Force
A force is a *push* or *pull* on an object.
Forces cause an object to accelerate…
To speed up
To slow down
To change direction
Unit: Newton (SI system)

Newton's First Law
The Law of Inertia.
A body in motion stays in motion at constant velocity and a body at rest stays at rest unless acted upon by an external force.
This law is commonly applied to the horizontal component of velocity, which is assumed not to change during the flight of a projectile.

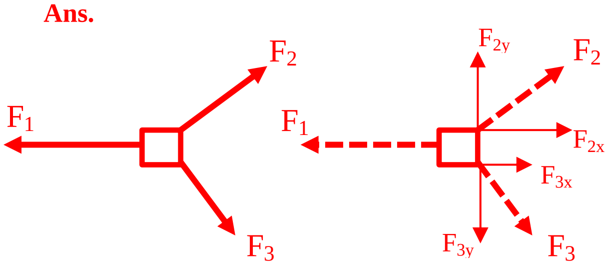
Problem: Newton's 1st Law (1998)

7. Three forces act on an object. If the object is in translational equilibrium, which of the following must be true?

I. The vector sum of the three forces must equal zero.
II. The magnitudes of the three forces must be equal
III. All three forces must be parallel

(A) I only
(B) II only
(C) I and III only
(D) II and III only
(E) I, II, and III

Explain your reasoning
Ans.

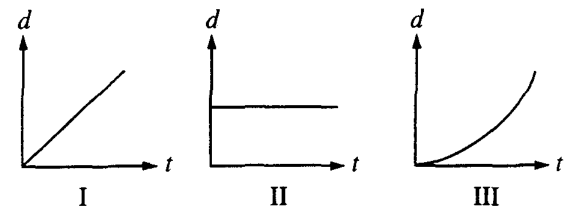


Equilibrium means that the sum of the forces in one dimension (left/right or up/down, for example) are equal to zero ($F_{net} = 0$ N). This means that the magnitudes of the *components of the forces* (F_x's and F_y's) have to cancel each other out but not the magnitude of the *forces* (F_1, F_2, and F_3) themselves. So $F_1 = F_{2x} + F_{3x}$ must be true and $F_{2y} = F_{3y}$. The forces definitely don't have to be parallel as can be seen in the figure above.

Problem: Newton's 1st Law (1998)

Questions 43-44

Three objects can only move along a straight, level path. The graphs below show the position d of each of the objects plotted as a function of time t.

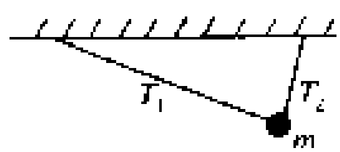


44. The sum of the forces on the object is zero in which of the cases?

(A) II only
(B) III only
(C) I and II only
(D) I and III only
(E) I, II, and III

Explain your reasoning
Ans. The straight line d vs. t graphs in I and II show that there is no acceleration (Forest Gump running). In graph III, the curve suggests that acceleration occurred (Usain Bolt running) since more distance is being covered per unit time. For acceleration to happen, according to Newton's Second Law, F = ma, and unbalanced force is applied to a mass causing that mass to accelerate. This only happens in graph III. In graphs I and II, any forces acting on the moving object must be canceling each other out because there is no acceleration occurring.

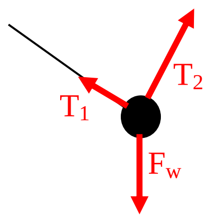
Problem: Newton's 1st Law (1984)



5. A hall of mass m is suspended from two strings of unequal length as shown above. The tensions T_1 and T_2 in the strings must satisfy which of the following relations?
(A) $T_1 = T_2$
(B) $T_1 > T_2$
(C) $T_1 < T_2$
(D) $T_1 + T_2 = mg$
(E) $T_1 - T_2 = mg$

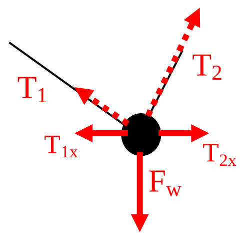
Show your work:

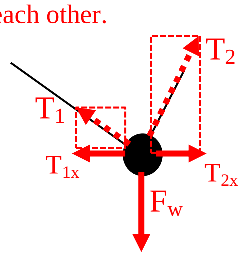
Ans. An easy way of looking at it is that String T_2 is more vertical than String T_1 and so is holding up more of the vertical weight of the ball, but just to make sure we should do a vector analysis of the forces at play.



There are three forces at play here. T_1, T_2, and the weight of the ball F_w.

The ball is in equilibrium so the forces up must equal the forces down and the forces left must equal the forces right. Let's deal with the forces left and right since we know they must be equal to each other.

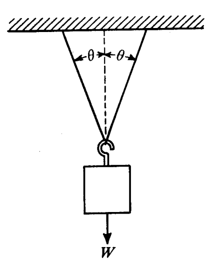




If we make our parallelograms around the forces T_{1x} and T_{2x} we have drawn, we can now see that the forces T_1 and T_2 are not equal to each other and that, in fact, T_2 is much greater than T_1

Problem: Newton's 1st Law (1988)

58. When an object of weight *W* is suspended from the center of a massless string as shown above, the tension at any point in the string is



(A) $2W\cos\theta$

(B) $\frac{W\cos\theta}{2}$

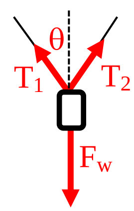
(C) $W\cos\theta$

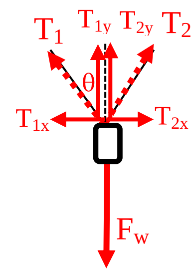
(D) $\frac{W}{2\cos\theta}$

(E) $\frac{W}{\cos\theta}$

Show your work:

Ans. The weight downwards must be supported by the two string strands at angles to each other above the weight. The force diagram is as follows:





In Equilibrium, the sum of the forces in the y-direction, $\Sigma F_y = 0 \rightarrow T_{1y} + T_{2y} = F_w$

T_{1y} and T_{2y} are the adjacent sides of the triangle shown above so we need to use cos θ to find the tension T_1 or T_2.

$\cos\theta = \frac{adj.}{hyp.} \rightarrow \cos\theta = \frac{T_{1y}}{T_1} \rightarrow T_1\cos\theta = T_{1y}$

$\rightarrow T_1 = \frac{T_{1y}}{\cos\theta} \rightarrow$ since $T_{1y} = ½F_w \rightarrow T_1 = \frac{\frac{1}{2}F_w}{\cos\theta}$

Since there is really only one string,

$\mathbf{T} = T_1 + T_2 = = \frac{T_{1y}}{\cos\theta} + \frac{T_{2y}}{\cos\theta}\ \frac{\frac{1}{2}F_w}{\cos\theta} + \frac{\frac{1}{2}F_w}{\cos\theta} = \frac{\mathbf{F_w}}{\mathbf{\cos\theta}}$

<u>Newton's Second Law</u>

A body accelerates when acted upon by a net external force.

The acceleration is proportional to the net force and is in the direction which the net force acts. This law is commonly applied to the vertical component of velocity.

Newton's Second Law

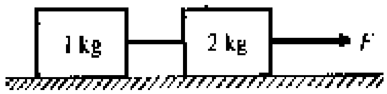
$\Sigma\mathbf{F} = m\mathbf{a}$

where $\Sigma\mathbf{F}$ is the net force measured in Newtons (N)
m is mass (kg)
a is acceleration (m/s^2)

<u>General Procedure for Solving Second Law Problems</u>

Step 1: Draw the problem
Step 2: Free Body Diagram
Step 3: Set up equations
$\Sigma\mathbf{F} = m\mathbf{a}$, $\Sigma F_x = ma_x$, $\Sigma F_y = ma_y$
Step 4: Substitute
Make a list of givens from the word problem.
Substitute in what you know.
Step 5: Solve

Problem: Second Law (1984)

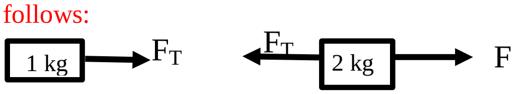


11. When the frictionless system shown above is accelerated by an applied force of magnitude *F*, the tension in the string between the blocks is
(A) $2F$
(B) F
(C) $\frac{2}{3}F$
(D) $\frac{1}{2}F$
(E) $\frac{1}{3}F$

Show your work:

Ans.

The setup can be simplified with a force diagrams as follows:



From the first force diagram we can see that:

$F_T = m_1 a$ → The tension equals the mass × acceler. of the 1 kg mass.

$F_T = 1\text{ kg} \cdot a = 1 \cdot a$ → subst. the mass of the block

From the second force diagram we can see that:

$F - F_T = m_2 a \rightarrow F - F_T = 2\text{ kg} \cdot a \rightarrow F - F_T = 2 \cdot a$

Substituting:

$F - F_T = 2(\mathbf{1 \cdot a}) \rightarrow F - F_T = 2 \cdot \mathbf{F_T}$

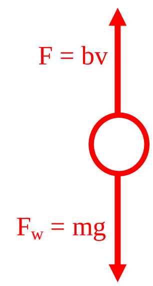
$F = 3F_T$

$\mathbf{\frac{1}{3}F = F_T}$

Problem: Second Law (1993)

2. A ball falls straight down through the air under the influence of gravity. There is a retarding force F on the ball with magnitude given by F = bv, where v is the speed of the ball and b is a positive constant. The magnitude of the acceleration a of the ball at any time is equal to which of the following?

(A) $g - b$
(B) $g - \frac{bv}{m}$
(C) $g + \frac{bv}{m}$
(D) $\frac{g}{b}$
(E) $\frac{bv}{m}$



Show your work:

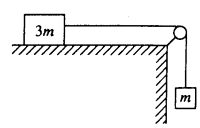
Ans. The force diagram on the ball is as shown above. The net force acting on the ball is the difference between the forces downwards and the forces upwards:

$\sum F_y = m \cdot a = F_w - F = m \cdot g - b \cdot v$

So $m \cdot a = m \cdot g - b \cdot v$

$\mathbf{a} = \frac{mg - bv}{m} = \mathbf{g - \frac{bv}{m}}$

Problem: Second Law (1993)



45. A block of mass 3m can move without friction on a horizontal table. This block is attached to another block of mass m by a cord that passes over a frictionless pulley, as shown above. If the masses of the cord and the pulley are negligible, what is the magnitude of the acceleration of the descending block?

(A) Zero
(B) g/4
(C) g/3
(D) 2g/3
(E) g

Show your work:

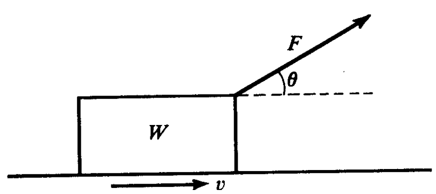
Ans. As in **Probl. 11**, we can define F_T in terms of what is being pulled by the cord.
F_T = **3m·a**
But the tension is also what is being pulled in the other direction by the hanging mass:
m·g = 3m·a
Solving for a: $\frac{g}{3} = a$

Newton's Third Law
For every action there exists an equal and opposite reaction.
If A exerts a force **F** on B, then B exerts a force of -**F** on A.

Weight
The force due to gravitation attraction.
W = mg

Normal Force
Force that prevents objects from penetrating each other
Reaction to other forces
Commonly a reaction to gravity

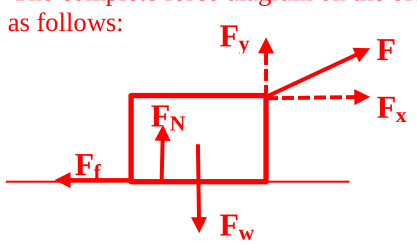
Problem: Normal Force Flat (1988)



4. A block of weight W is pulled along a horizontal surface at constant speed v by a force F. which acts at an angle of θ with the horizontal, as shown above. The normal force exerted on the block by the surface has magnitude

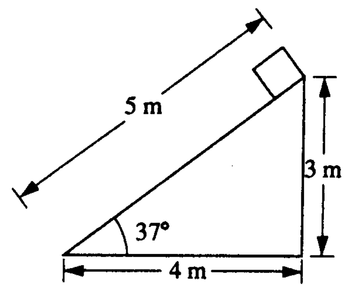
(A) W - Fcos θ
(B) W - Fsin θ
(C) W
(D) W + Fsin θ
(E) W + Fcos θ

Show your work
Ans. The complete force diagram on the block is as follows:



Because the block is not elevating vertically or crashing downwards through the ground, it must be in equilibrium vertically. This means the forces upwards equal the forces downwards.
$\sum F_y = m \cdot a = 0$
$\mathbf{F\uparrow = F\downarrow}$
$\mathbf{F_N + F_y = F_w} \rightarrow F_N = F_w - F_y \rightarrow F_N = \mathbf{W} - F_y$
F_y can be expressed in terms of F through right triangle trigonometry (F_y is the opposite side of the triangle):
$F_y = F \cdot \sin\theta$
Substituting: $F_N = W - \mathbf{F \cdot \sin\theta}$

Problem: Normal Force Ramp (1993)

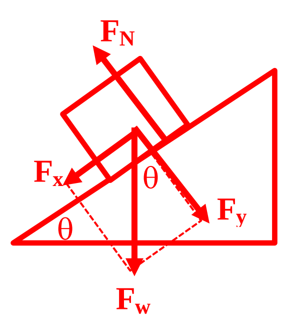


A plane 5 meters in length is inclined at an angle of 37°, as shown above. A block of weight 20 newtons is placed at the top of the plane and allowed to slide down.

62. The magnitude of the normal force exerted on the block by the plane is most nearly

(A) l0 N
(B) 12N
(C) l6 N
(D) 20 N
(E) 33 N

Show your work
Ans. The force diagram on the block on the inclined plane is shown below:



The angle of the force triangle is also 37° by similar triangles.

Because of equilibrium ($\sum F_y = m \cdot a = 0$), the normal force, F_N, is equal and opposite to F_y, the vertical component of the block's weight, F_w.

F_y is the *adjacent* side of the force triangle shown so we will use cos θ:

$F_y = F_w \cdot \cos\theta = F_w \cdot \frac{adj.}{hyp.}$

Substituting our given values:

$F_y = \mathbf{20\ N} \cdot \frac{\mathbf{4\ m}}{\mathbf{5\ m}} = 20 \cdot \mathbf{0.8} = \mathbf{16}$

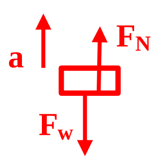
Problem: Elevators and Normal Force (PAB)

2. A 50-kg middle school student stands on a scale in an elevator that is moving downward, but slowing with an acceleration of magnitude 2.0 m/s^2. What does the scale read (in N)?

a) 300
b) 400
c) 500
d) 600
e) 700

Show your work

Ans. The forces on the scale are shown below:



The scale will read the difference between the forces upwards and downwards:

$\sum F_y = m \cdot a = F_N - F_w$

Substituting our givens:

$50\ kg \cdot 2\ m/s^2 = F_N - 50\ kg \cdot 10\ m/s^2$

$100 = F_N - 500$

$\mathbf{600 = F_N}$

Friction

The force that opposes a sliding motion.
Enables us to walk, drive a car, etc.
Due to microscopic irregularities in even the smoothest of surfaces.
There are two types of friction
Static friction exists before sliding occurs
Kinetic friction exists after sliding occurs
In general *Kinetic friction* <= *Static friction*

Friction and the Normal Force

The frictional force which exists between two surfaces is directly proportional to the normal force.
That's why friction on a sloping surface is less than friction on a flat surface.

Calculating Static Friction

$f_s \le \mu_s N$

f_s : static frictional force (N)
μ_s: coefficient of static friction
N: normal force (N)

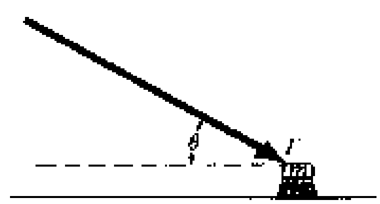
Static friction increases as the force trying to push an object increases… up to a point!

Calculating Kinetic Friction

$f_k = \mu_k N$

f_k : kinetic frictional force (N)
μ_k: coefficient of kinetic friction
N: normal force (N)

Problems: Friction on Flat Surface (1984)

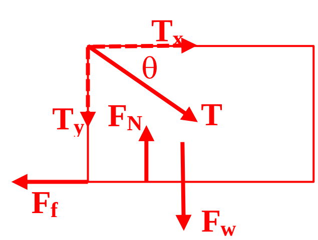


61. A push broom of mass m is pushed across a rough horizontal floor by a force of magnitude *T* directed at angle *θ* as shown above. The coefficient of friction between the broom and the floor is μ. The frictional force on the broom has magnitude

(A) $\mu(mg - T\sin\theta)$
(B) $\mu(mg + T\sin\theta)$
(C) $\mu(mg + T\cos\theta)$
(D) $\mu(mg - T\cos\theta)$
(E) μmg

Show your work

Ans. the force diagram on the push broom head is show below:

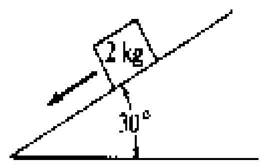


The force of friction is given by $F_f = \mu \cdot F_N$.
F_N is the only upwards vertical force. Because the push broom is in equilibrium in the y-direction,
$\sum F_y = m \cdot a = 0$
$\mathbf{F\uparrow = F\downarrow}$
$\mathbf{F_N = F_w + T_y \rightarrow}$
F_y can be expressed in terms of F, the pushing force, through right triangle trigonometry (F_y is the opposite side of the triangle):
$T_y = T\cdot \sin\theta$
And the weight: $F_w = m\cdot g$
Substituting: $F_N = \mathbf{m\cdot g + T\cdot \sin\theta}$
Substituting into the friction formula:
$F_f = \mu\cdot F_N = \mu\cdot(\mathbf{m\cdot g + T\cdot \sin\theta})$

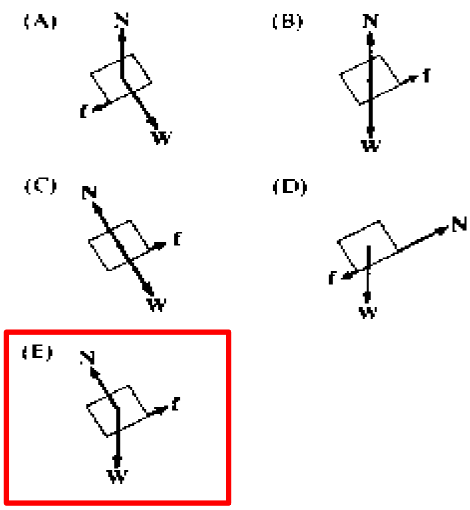
Problems: Friction on Ramp(1984)

Questions 6-7

A 2-kilogram block slides down a 30° incline as shown above with an acceleration of 2 meters per second squared.



6. Which of the following diagrams best represents the gravitational force **W**, the frictional force **f**, and the normal force **N** that act on the block?



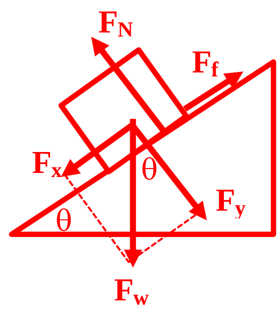
Ans. The force of weight W points down to the center of the earth so this eliminates answer choice (A). The Normal force, N, is perpendicular to the surface the block is on, so this eliminates Answer choices (B) and (C). Friction, f, always opposes the motion so this also eliminates answers (B) and (C). Only **Ans. E works.**

7. The magnitude of the frictional force along the plane is most nearly
(A) 2.5 N
(B) 5 N
(C) **6 N**
(D) 10 N
(E) 16 N

Show your work:

Ans. The block is *not* in equilibrium in the x-direction along the inclined plane because the block is accelerating downwards.

This means that $\sum F_x = m\cdot a \neq 0$
$\sum F_x = m\cdot a = F_x - F_f$ where F_x is the component of the weight of the block, F_w, along the inclined plane (see force diagram below).



F_x is the opposite side of the right triangle shown so $F_x = F_W \cdot \sin\theta = m\cdot g \cdot \sin\theta$
$m\cdot a = F_x - F_f$

Substituting:
$m\cdot a = \mathbf{m\cdot g \cdot \sin\theta} - F_f$
$m\cdot a - \mathbf{m\cdot g\cdot \sin\theta} = F_f$

Substituting your givens:
$2\text{ kg}\cdot(2\text{ m/s}^2) - 2\text{ kg}\cdot 10\text{ m/s}^2\cdot \sin 30 = F_f$
$4 - 20\cdot(½) = F_f$
$4 - 10 = F_f$
$\mathbf{-6\text{ N} = F_f}$