Physics C Mechanics Summer Assignment *Answer Key*

There will be a test covering the summer assignment the first week of class.

1. The following are ordinary physics problems. Place the answer in scientific notation when appropriate and simplify the units (Scientific notation is used when it takes less time to write than the ordinary number does. As an example 200 is easier to write than 2.00×10^2, but 2.00×10^8 is easier to write than 200,000,000). Do your best to cancel units, and attempt to show the simplified units in the final answer.

a. $T_s = 2\pi\sqrt{\frac{4.5\times10^{-2}\,kg}{2.0\times10^{3}\,kg/s^2}} = 2\pi\sqrt{2.25\times10^{-5}kg \div \frac{kg}{s^2}} = 2\pi\sqrt{2.25\times10^{-5}kg\cdot\frac{s^2}{kg}}$ $=2\pi\sqrt{2.25\times10^{-5}s^2} = 2.98\times10^{-2}$ s

2.98×10^{-2} s (or .0298 s)

b. $K = ½(6.6\times10^2\text{ kg})(2.11\times10^4\text{ m/s})^2$

$= ½(6.6\times10^2\text{ kg})(4.4521\times10^8\text{ m}^2/\text{s}^2) = 1.47\times10^{11}\text{ kg m}^2/\text{s}^2$

$1.47\times10^{11}\text{ kg m}^2/\text{s}^2$

c. $F = \left(9.0\times10^9\,\frac{N\cdot m^2}{C^2}\right)\frac{(3.2\times10^{-9}C)(9.6\times10^{-9}C)}{(0.32m)^2} =$

$=(9.0\times10^9\,\frac{N\cdot m^2}{C^2})\frac{(3.2\times10^{-9}\,C)(9.6\times10^{-9}\,C)}{1.024\times10^{-1}\,m^2} = \frac{2.7648\times10^{-7}\,N\cdot C^2\cdot m^2}{1.024\times10^{-1}\,C^2\cdot m^2} = 2.7\times10^{-6}\,N$

$2.7\times10^{-6}\,N$

d. $K_{max} = \left(6.63\times10^{-34}\,\frac{J}{s}\right)(7.09\times10^{14}s) - 2.17\times10^{-19}J =$

$= 4.70067\times10^{-19}\,J\cdot\frac{s}{s} - 2.17\times10^{-19}\,J = 2.53067\times10^{-19}\,J$

$2.53067\times10^{-19}\,J$

e. $e = \frac{1.7\times10^3J - 3.3\times10^2J}{1.7\times10^3J} = \frac{1.37\times10^3J}{1.7\times10^3J} = \frac{1.37}{1.7} = 8.0588$

8.0588

f. $1.33\sin 25.0° = 1.5\sin\theta \rightarrow 5.620822\times10^{-1} = 1.5\sin\theta$

$\frac{5.620822\times10^{-1}}{1.5} = \sin\theta \rightarrow sin^{-1}\left(\frac{5.620822\times10^{-1}}{1.5}\right) = sin^{-1}(\sin\theta)$

$\rightarrow sin^{-1}(3.74721\times10^{-1}) = \theta \rightarrow 2.2007\times10^1 = \theta$

$22° = \theta$

2. Often problems on the AP exam are done with variables only. Solve for the variable indicated. Don't let the different letters confuse you. Manipulate them algebraically as though they were numbers.

a. $K = \frac{1}{2}kx^2$, $x = \sqrt{\frac{2\cdot K}{k}}$

$\rightarrow \frac{2}{1}\cdot K = \frac{1}{2}kx^2\cdot\frac{2}{1} \rightarrow 2\cdot K = kx^2 \rightarrow \frac{2\cdot K}{k} = \frac{kx^2}{k}$

$\rightarrow \frac{2\cdot K}{k} = x^2 \rightarrow \sqrt{\frac{2\cdot K}{k}} = \sqrt{x^2} \rightarrow \sqrt{\frac{2\cdot K}{k}} = x$

b. $T_p = 2\pi\sqrt{\frac{\ell}{g}}$, $g = \frac{l\cdot 4\pi^2}{T_P{}^2}$

$\rightarrow \frac{T_P}{2\pi} = \frac{2\pi}{2\pi}\sqrt{\frac{l}{g}} \rightarrow \frac{T_P}{2\pi} = \sqrt{\frac{l}{g}} \rightarrow \left(\frac{T_P}{2\pi}\right)^2 = \left(\sqrt{\frac{l}{g}}\right)^2$

$\rightarrow \frac{T_P{}^2}{2^2\pi^2} = \frac{l}{g} \rightarrow \frac{T_P{}^2}{4\pi^2} = \frac{l}{g} \rightarrow T_P{}^2\cdot\boldsymbol{g} = \boldsymbol{l}\cdot 4\pi^2$

$\rightarrow \frac{T_P{}^2\cdot g}{T_P{}^2} = \frac{l\cdot 4\pi^2}{T_P{}^2} \rightarrow g = \frac{l\cdot 4\pi^2}{T_P{}^2}$

c. $F_g = G\frac{m_1m_2}{r^2}$, $r = \sqrt{G\frac{m_1m_2}{F_g}}$

$\rightarrow \frac{r^2}{1}\cdot F_g = G\frac{m_1m_2}{r^2}\cdot\frac{r^2}{1} \rightarrow F_g\cdot\boldsymbol{r^2} = G\cdot m_1m_2 \rightarrow$

$\rightarrow \frac{F_g\cdot r^2}{F_g} = \frac{G\cdot m_1m_2}{F_g} \rightarrow r^2 = G\frac{m_1m_2}{F_g} \rightarrow$

$\sqrt{r^2} = \sqrt{G\frac{m_1m_2}{F_g}} \rightarrow \mathbf{r} = \sqrt{G\frac{m_1m_2}{F_g}}$

d. $mgh = \frac{1}{2}mv^2$, $v = \sqrt{2gh}$

$\rightarrow \frac{2}{1}\cdot mgh = \frac{1}{2}mv^2\cdot\frac{2}{1} \rightarrow 2mgh = mv^2$

$\rightarrow \frac{2mgh}{m} = \frac{mv^2}{m} \rightarrow 2gh = v^2 \rightarrow \sqrt{2gh} = \sqrt{v^2}$

$\rightarrow \sqrt{2gh} = \boldsymbol{v}$

e. $x = x_o + v_o t + \frac{1}{2}at^2$ $,t = \frac{-v_o \pm \sqrt{v_o^2 - 2a(x_o - x)}}{a}$

$\rightarrow$ $0 = x_o - x + v_o \cdot t + \frac{1}{2}a \cdot t^2$ $\rightarrow$ use quadratic formula

$0 = c + b \cdot x + a \cdot x^2$

$\rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow t = \frac{-v_o \pm \sqrt{v_o^2 - 4(\frac{1}{2}a)(x_o - x)}}{2(\frac{1}{2}a)} \rightarrow$

$t = \frac{-v_o \pm \sqrt{v_o^2 - 2a(x_o - x)}}{a}$

f. $B = \frac{\mu_o}{2\pi}\frac{I}{r}$ $,r = \frac{\mu_o I}{2\pi B}$

$\rightarrow \frac{r}{1} \cdot B = \frac{\mu_o I}{2\pi r} \cdot \frac{r}{1} \rightarrow B \cdot r = \frac{\mu_o I}{2\pi} \rightarrow$

$\rightarrow \frac{1}{B} \cdot Br = \frac{\mu_o I}{2\pi} \cdot \frac{1}{B} \rightarrow r = \frac{\mu_o I}{2\pi B} \rightarrow$

g. $x_m = \frac{m\lambda L}{d}$ $,d = \frac{m\lambda L}{x_m}$

$\rightarrow \frac{d}{1} \cdot x_m = \frac{m\lambda L}{d} \cdot \frac{d}{1} \rightarrow x_m \cdot d = m\lambda L \rightarrow$

$\rightarrow \frac{x_m \cdot d}{x_m} = \frac{m\lambda L}{x_m} \rightarrow d = \frac{m\lambda L}{x_m}$

h. $pV = nRT$ $,T = \frac{pV}{nR}$

$\rightarrow \frac{pV}{nR} = \frac{nRT}{nR} \rightarrow \frac{pV}{nR} = T$

i. $\sin\theta_c = \frac{n_1}{n_2}$ $,\theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right)$

$\rightarrow \sin^{-1}(\sin\theta_c) = \sin^{-1}\left(\frac{n_1}{n_2}\right) \rightarrow \theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right)$

j. $qV = \frac{1}{2}mv^2$ $,v = \sqrt{\frac{2qV}{m}}$

$\rightarrow \frac{2}{1} \cdot qV = \frac{1}{2}mv^2 \cdot \frac{2}{1} \rightarrow 2qV = mv^2$

$\rightarrow \frac{2qV}{m} = \frac{mv^2}{m} \rightarrow \frac{2qV}{m} = v^2 \rightarrow \sqrt{\frac{2qV}{m}} = \sqrt{v^2}$

$\rightarrow \sqrt{\frac{2qV}{m}} = v$

k. $\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$ $,s_i = \frac{fs_o}{s_o + f}$

$\rightarrow \frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} \rightarrow \frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} \rightarrow$ isolate $\frac{1}{s_i}$

$-\frac{1}{s_o} \quad -\frac{1}{s_o}$

$\rightarrow \frac{1}{f} - \frac{1}{s_o} = \frac{1}{s_i} \rightarrow$ find a common denominator

$\rightarrow \left(\frac{fs_o}{fs_o}\right) \cdot \frac{1}{f} - \left(\frac{fs_o}{fs_o}\right) \cdot \frac{1}{s_o} = \frac{1}{s_i}$

$\rightarrow \frac{s_o}{fs_o} + \frac{f}{fs_o} = \frac{1}{s_i}$

$\rightarrow \frac{s_o + f}{fs_o} = \frac{1}{s_i} \rightarrow$ cross multiply

$\rightarrow s_i(s_o + f) = fs_o \rightarrow \frac{s_i(s_o + f)}{s_o + f} = \frac{fs_o}{s_o + f}$

$\rightarrow s_i = \frac{fs_o}{s_o + f}$

Science uses the ***KMS*** system (***SI***: System Internationale). ***KMS*** stands for kilogram, meter, second. These are the units of choice of physics. The equations in physics depend on unit agreement. So you must convert to ***KMS*** in most problems to arrive at the correct answer.

kilometers (*km*) to meters (*m*)	and meters to kilometers	gram (*g*) to kilogram (*kg*)
centimeters (*cm*) to meters (*m*)	and meters to centimeters	Celsius (^oC) to Kelvin (*K*)
millimeters (*mm*) to meters (*m*)	and meters to millimeters	atmospheres (*atm*) to Pascals (*Pa*)
nanometers (*nm*) to meters (*m*)	and metes to nanometers	liters (*L*) to cubic meters (m^3)
micrometers (μm) to meters (*m*)		

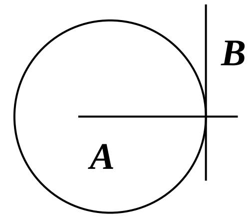
Other conversions will be taught as they become necessary.

3. What if you don't know the conversion factors? Colleges want students who can find their own information (so do employers). **Hint:** Try a good dictionary and look under "measure" or "measurement". Or the Internet? Enjoy.

a. 4008 *g* = $4008\ g \times \frac{1\ kg}{1000\ g} = 4.008\ kg$	**h.** 25.0 μm = $25.0\ \mu m \times \frac{1\ m}{1\times10^6\ \mu m} = 2.5 \times 10^{-5}\ m$
b. 1.2 *km* = $1.2\ km \times \frac{1000\ m}{1\ km} = 1200\ m$	**i.** 2.65 *mm* = $2.65\ mm \times \frac{1\ m}{1000\ mm} = 0.00265\ m$
c. 823 *nm* = $823\ nm \times \frac{1\ m}{1\times10^9\ nm} = 8.23 \times 10^{-7}\ m$	**j.** 8.23 *m* = $8.23\ m \times \frac{1\ km}{1000\ m} = 0.00823\ km$
d. 298 *K* = $298\ K - 273 = 25^o C$	**k.** 5.4 *L* = $5.4\ L \times \frac{0.001\ m^3}{1\ L} = 0.0054\ m^3$
e. 0.77 *m* = $0.77\ m \times \frac{100\ cm}{1\ m} = 77\ cm$	**l.** 40.0 *cm* = $40.0\ cm \times \frac{1\ m}{100\ cm} = 0.4\ m$
f. 8.8x10^{-8} *m* = $8.8 \times 10^{-8}\ m \times \frac{1000\ mm}{1\ m} = 8.8\times10^{-5}\ mm$	**m.** 6.23 x 10^{-7} *m* = $6.23 \times 10^{-7}\ m \times \frac{1\times10^9\ nm}{1\ m} = 623\ nm$
g. 1.2 *atm* = $1.2\ atm \times \frac{101,325\ Pa}{1\ atm} = 121,590\ Pa$	**n.** 1.5 x 10^{11} *m* = $1.5\times10^{11}\ m \times \frac{1\ km}{1000\ m} = 1.5\times10^8\ km$

4. Many of the AP Physics C problems assume a working knowledge of geometry and trigonometry to solve idealized mechanics problems. Solve the following geometric problems.

a. Line ***B*** touches the circle at a single point. Line ***A*** extends through the center of the circle.



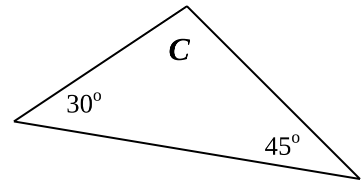
i. What is line ***B*** in reference to the circle?

The tangent line (a tangent line touches a shape in exactly one point)

ii. How large is the angle between lines ***A*** and ***B***?

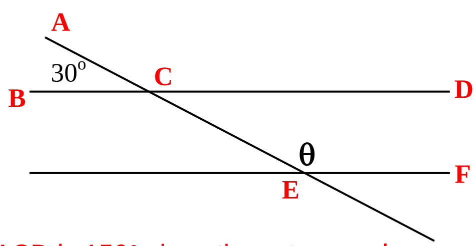
Line segments A and B are perpendicular so the angle is 90°

b. What is angle ***C***?



The sum of the angles of a triangle = 180° so ∠C = 180° – 30° – 45°

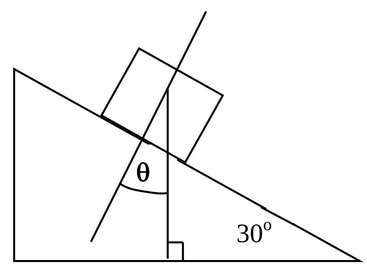
∠C = **105°**



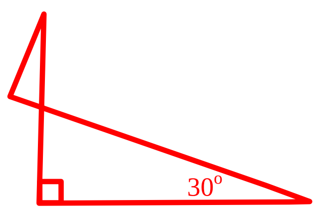
c. What is angle θ ?

If ∠ABC is 30°, then ∠ACD is 150° since these two angles are supplementary. ∠CEF must also = 150° since corresponding angles are congruent.

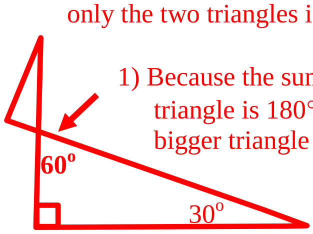
d. How large is *θ*?



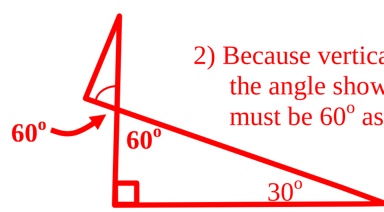
Angle θ = 30°.



Ans. This is basically a geometry problem. If we get rid of all the other stuff and leave only the two triangles in question, we have the figure above.

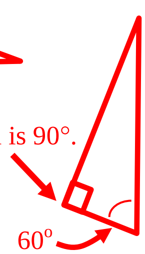


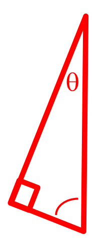
1) Because the sum of the angles of a triangle is 180°, the missing angle of the bigger triangle must be 60°



2) Because vertical angles are congruent, the angle shown of the smaller triangle must be 60° as well.

3) The angle shown is 90°.





4) That makes the angle θ, shown 30, just like the initial given angle.

e. The radius of a circle is 5.5 *cm*,

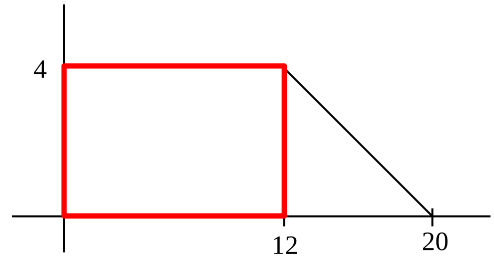
i. What is the circumference in meters? **1.1π m**

Ans. $C = 2\pi r = 2\pi(5.5\text{ cm}) = 110\pi\text{ cm} \times \frac{1\ m}{100\ cm} =$ **1.1π m**

ii. What is its area in square meters? $0.003\pi\ m^2$ __

Ans. 5.5 cm $\times \frac{1\ m}{100\ cm} = 0.055\ m$

$A = \pi r^2 = \pi(0.055\ m)^2 =$ **$0.003\pi\ m^2$**



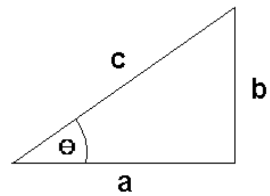
f. What is the area under the curve at the right?__**$64\ un^2$**__

Ans. The area under the curve at right is basically the area of two shapes, a rectangle and a triangle. You just need to add them.

$A_{\square} = l \times w = 12 \times 4 = 48\ un^2$ **$A_{\Delta} = ½bh = ½(8)(4) = 16\ un^2$**

$48\ un^2 + 16\ un^2 = 64\ un^2$

Using the generic triangle to the right, Right Triangle Trigonometry and Pythagorean Theorem solve the following. ***Your calculator must be in degree mode.***



a. θ = 55° and ***c*** = 32 *m*, solve for ***a*** and ***b***. _______________

Ans. $\sin\theta = \frac{opp.}{hyp.} \rightarrow \sin 55° = \frac{b}{32\ m} \rightarrow 32 \cdot \sin 55° = b \rightarrow \mathbf{26.21\ m = b}$

$\cos\theta = \frac{adj.}{hyp.} \rightarrow \cos 55° = \frac{a}{32\ m} \rightarrow 32 \cdot \cos 55° = a \rightarrow \mathbf{18.35\ m = a}$

b. θ = 45° and ***a*** = 15 *m/s*, solve for ***b*** and ***c***._______________

Ans. $\cos\theta = \frac{adj.}{hyp.} \rightarrow \cos\theta = \frac{a}{c} \rightarrow c \cdot \cos 45° = 15\ m/s \rightarrow c = \frac{15}{\cos 45} \rightarrow \mathbf{c = 21.21\ m}$

$a^2 + b^2 = c^2 \rightarrow b^2 = c^2 - a^2 \rightarrow b = \sqrt{c^2 - a^2} \rightarrow b = \sqrt{21.21^2 - 15^2} \rightarrow b = \sqrt{225}\ \mathbf{= 15\frac{m}{s}}$

c. ***b*** = 17.8 *m* and θ = 65°, solve for ***a*** and ***c***. _______________

Ans. $\sin\theta = \frac{opp.}{hyp.} \rightarrow \sin\theta = \frac{b}{c} \rightarrow c \cdot \sin 65° = 17.8\ m \rightarrow c = \frac{17.8}{\sin 65} \rightarrow \mathbf{c = 19.64\ m}$

$a^2 + b^2 = c^2 \rightarrow a^2 = c^2 - b^2 \rightarrow a = \sqrt{c^2 - b^2} \rightarrow b = \sqrt{19.64^2 - 17.8^2} \rightarrow b = \sqrt{68.89}\ \mathbf{= 8.3\ m}$

d. ***a*** = 250 *m* and ***b*** = 180 *m*, solve for θ and ***c***. _______________

Ans. $a^2 + b^2 = c^2 \rightarrow \sqrt{a^2 + b^2} = c \rightarrow \sqrt{(250\ m)^2 + (180\ m)^2} = \sqrt{308.06\ m} = c \rightarrow \mathbf{17.55\ m = c}$

$\tan\theta = \frac{opp.}{adj.} \rightarrow \tan\theta = \frac{b}{a} \rightarrow \tan\theta = \frac{250\ m}{180\ m} \rightarrow tan^{-1}(\tan\theta = \frac{250}{180}) \rightarrow \theta = tan^{-1}(1.388 \ldots) \rightarrow \mathbf{\theta = 54.25°}$

e. ***a*** =25 *cm* and ***c*** = 32 *cm*, solve for ***b*** and θ._______________

Ans. $a^2 + b^2 = c^2 \rightarrow b^2 = c^2 - a^2 \rightarrow b = \sqrt{c^2 - a^2} \rightarrow b = \sqrt{(32\ cm)^2 - (25\ cm)^2} \rightarrow$

$b = \sqrt{399}\ \mathbf{= 19.98\ cm}$

$\tan\theta = \frac{opp.}{adj.} \rightarrow \tan\theta = \frac{b}{a} \rightarrow \tan\theta = \frac{19.98\ cm}{25\ cm} \rightarrow tan^{-1}(\tan\theta = \frac{19.98}{25}) \rightarrow \theta = tan^{-1}(0.798 \ldots) \rightarrow \mathbf{\theta = 38.62°}$

f. ***b*** =104 *cm* and ***c*** = 165 *cm*, solve for ***a*** and θ._______________

Ans. $a^2 + b^2 = c^2 \rightarrow a^2 = c^2 - b^2 \rightarrow a = \sqrt{c^2 - b^2} \rightarrow a = \sqrt{(165\ cm)^2 - (104\ cm)^2} \rightarrow$

$b = \sqrt{16{,}409}\ \mathbf{= 128.10\ cm}$

$\tan\theta = \frac{opp.}{adj.} \rightarrow \tan\theta = \frac{b}{a} \rightarrow \tan\theta = \frac{128.10\ cm}{104\ cm} \rightarrow tan^{-1}(\tan\theta = \frac{128.10}{104}) \rightarrow \theta = tan^{-1}(1.23 \ldots) \rightarrow \mathbf{\theta = 50.93°}$

Vectors

Most of the quantities in physics are vectors. ***This makes proficiency in vectors extremely important***.

Magnitude: Size or extent, the numerical value.

Direction: Alignment or orientation of any position with respect to any other position.

Scalars: A physical quantity described by a single number and units. A quantity described by **magnitude only**.

Examples: time, mass, and temperature

Vector: A physical quantity with **both a magnitude and a direction**. A directional quantity.

Examples: velocity, acceleration, force

Notation: $\vec{A}$ or $\xrightarrow{\vec{A}}$ — Length of the arrow is proportional to the vectors magnitude. Direction the arrow points is the direction of the vector.

Negative Vectors

Negative vectors have the same magnitude as their positive counterpart. They are just pointing in the opposite direction.

$\xrightarrow{\vec{A}}$ $\xleftarrow{-\vec{A}}$

Vector Addition and subtraction

Think of it as vector addition only. The result of adding vectors is called the resultant. $\vec{R}$

$\vec{A}+\vec{B}=\vec{R}$ $\xrightarrow{\vec{A}}$ + $\xrightarrow{\vec{B}}$ = $\xrightarrow{\quad\vec{R}\quad}$

So if ***A*** has a magnitude of 3 and ***B*** has a magnitude of 2, then ***R*** has a magnitude of 3+2=5.

When you need to subtract one vector from another think of the one being subtracted as being a negative vector. Then add them.

$\vec{A}-\vec{B}$ *is really* $\vec{A}+-\vec{B}=\vec{R}$ $\xrightarrow{\vec{A}}$ + $\xleftarrow{-\vec{B}}$ = $\xrightarrow{\vec{R}}$

A negative vector has the same length as its positive counterpart, but its direction is reversed. So if ***A*** has a magnitude of 3 and ***B*** has a magnitude of 2, then ***R*** has a magnitude of 3+(−2)=1.

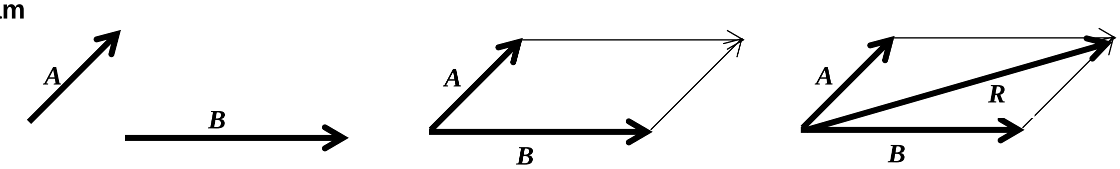
This is very important.

In physics a negative number does not always mean a smaller number. Mathematically –2 is smaller than +2, but in physics these numbers have the same magnitude (size), they just point in different directions (180^{o} apart).

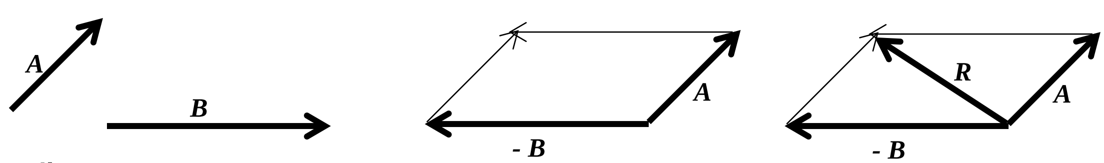
There are two methods of adding vectors

Parallelogram

A + ***B***

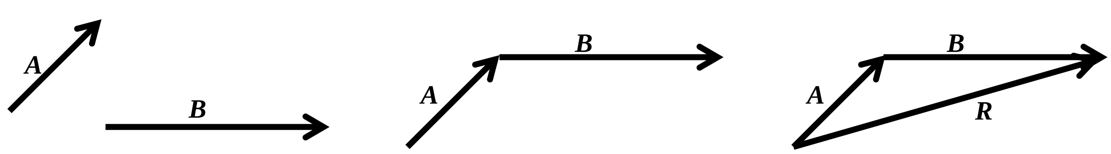


A – ***B***

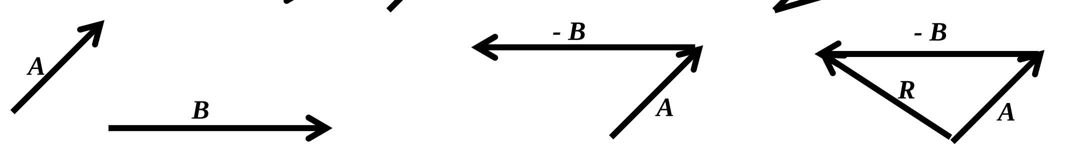


Tip (Head) to Tail

A + ***B***



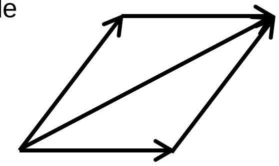
A – ***B***



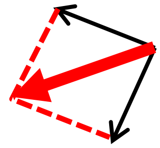
It is readily apparent that both methods arrive at the exact same solution since either method is essentially a parallelogram. It is useful to understand both systems. In some problems one method is advantageous, while in other problems the alternative method is superior.

5. Draw the resultant vector using the parallelogram method of vector addition.

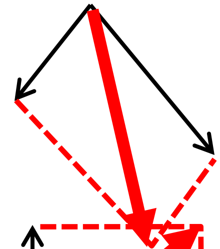
Example



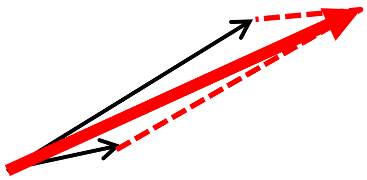
b.



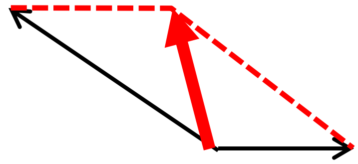
d.



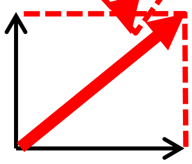
a.



c.

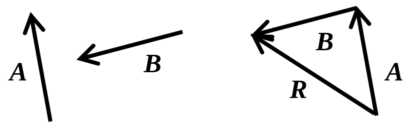


e.

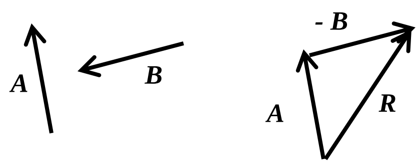


6. Draw the resultant vector using the tip to tail method of vector addition. Label the resultant as vector ***R***

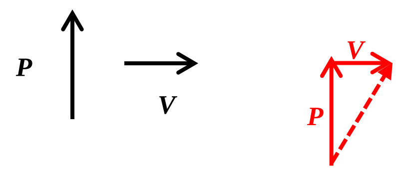
Example 1: ***A*** + ***B***



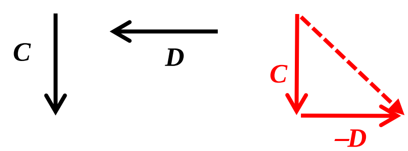
Example 2: ***A*** – ***B***



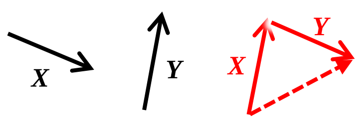
c. ***P*** + ***V***



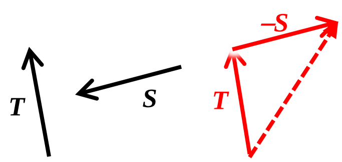
d. ***C*** – ***D***



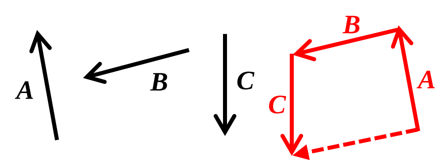
a. ***X*** + ***Y***



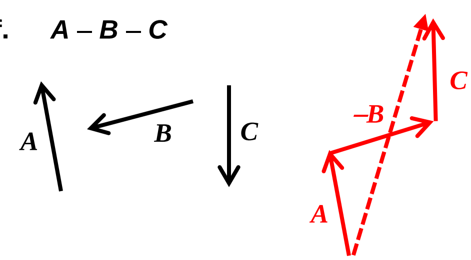
b. ***T*** – ***S***



e. ***A*** + ***B*** + ***C***

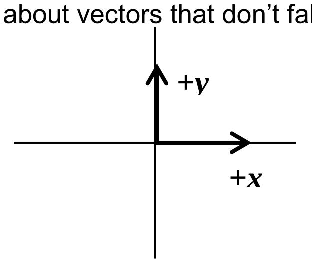


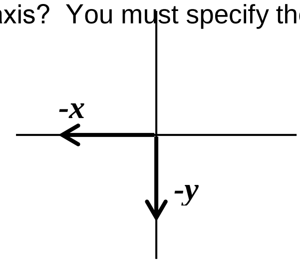
f. ***A*** – ***B*** – ***C***

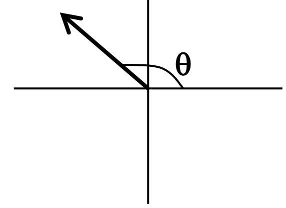


Direction: What does positive or negative direction mean? How is it referenced? The answer is the coordinate axis system. **In physics a coordinate axis system is used to give a problem a frame of reference.** Positive direction is a vector moving in the positive ***x*** or positive ***y*** direction, while a negative vector moves in the negative ***x*** or negative ***y*** direction (This also applies to the ***z*** direction, which will be used sparingly in this course).

What about vectors that don't fall on the axis? You must specify their direction using degrees measured from East.



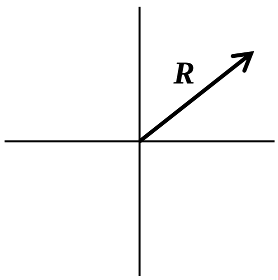


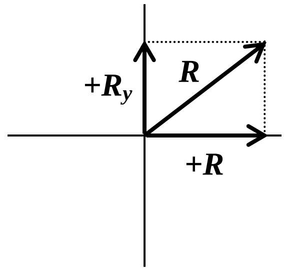


Component Vectors

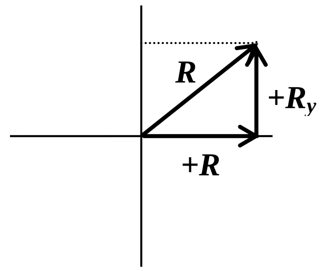
A resultant vector is a vector resulting from the sum of two or more other vectors. Mathematically the resultant has the same magnitude and direction as the total of the vectors that compose the resultant. Could a vector be described by two or more other vectors? Would they have the same total result?

This is the reverse of finding the resultant. You are given the resultant and must find the component vectors on the coordinate axis that describe the resultant.





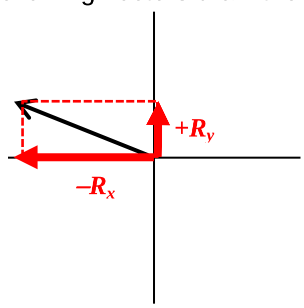
or



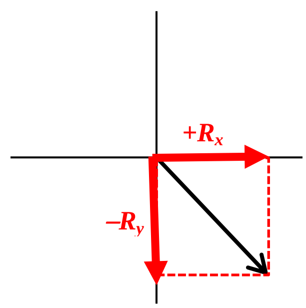
Any vector can be described by an ***x*** axis vector and a ***y*** axis vector which summed together mean the exact same thing. The advantage is you can then use plus and minus signs for direction instead of the angle.

7. For the following vectors draw the component vectors along the ***x*** and ***y*** axis.

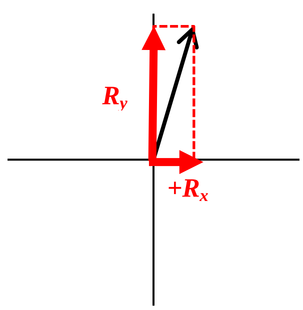
a.



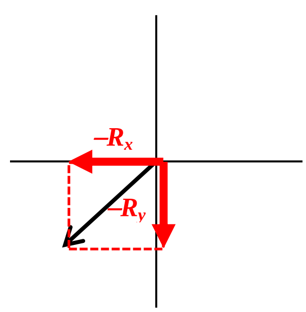
c.



b.



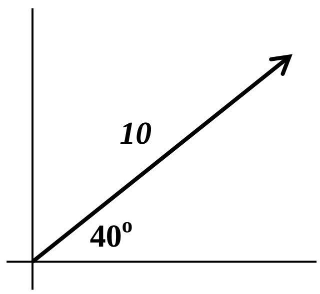
d.

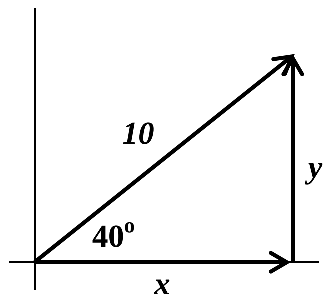


Obviously the quadrant that a vector is in determines the sign of the ***x*** and ***y*** component vectors.

Trigonometry and Vectors

Given a vector, you can now draw the ***x*** and ***y*** component vectors. The sum of vectors ***x*** and ***y*** describe the vector exactly. Again, any math done with the component vectors will be as valid as with the original vector. The advantage is that math on the ***x*** and/or ***y*** axis is greatly simplified since direction can be specified with plus and minus signs instead of degrees. But, how do you mathematically find the length of the component vectors? Use trigonometry.





$$\cos\theta = \frac{adj}{hyp} \qquad \sin\theta = \frac{opp}{hyp}$$

$$adj = hyp\cos\theta \qquad opp = hyp\sin\theta$$

$$x = hyp\cos\theta \qquad y = hyp\sin\theta$$

$$x = 10\cos 40^o \qquad y = 10\sin 40^o$$

$$x = 7.66 \qquad y = 6.43$$

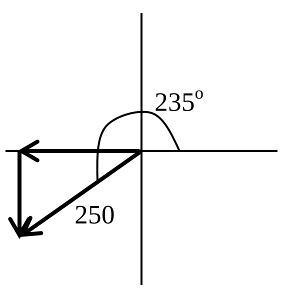
8. Solve the following problems. You will be converting from a polar vector, where direction is specified in ***<u>degrees measured counterclockwise from east</u>***, to component vectors along the ***x*** and ***y*** axis. Remember the plus and minus signs on you answers. They correspond with the quadrant the original vector is in.

   Hint: Draw the vector first to help you see the quadrant. Anticipate the sign on the ***x*** and ***y*** vectors. Do not bother to change the angle to less than 90°. Using the number given will result in the correct + and – signs.

   The first number will be the magnitude (length of the vector) and the second the degrees from east.

   <u>Your calculator must be in degree mode.</u>

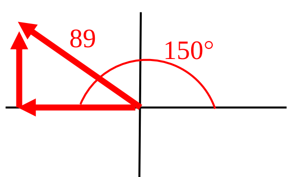
   Example: 250 at 235°



$$x = hyp\cos\theta$$
$$x = 250\cos 235^o$$
$$x = -143$$

$$y = hyp\sin\theta$$
$$y = 250\sin 235^o$$
$$y = -205$$

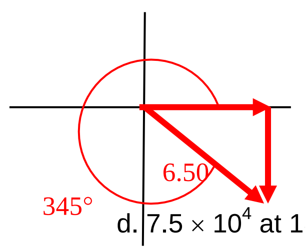
a. 89 at 150°



x = hyp cos θ
x = 89 cos 150°
x = –77.08

y = hyp sin θ
x = 89 sin 150°
x = 44.5

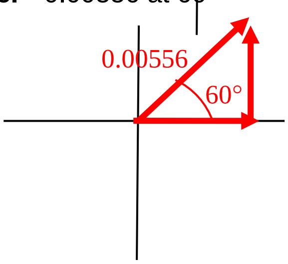
b. 6.50 at 345°



x = hyp cos θ
x = 6.50 cos 345°
x = 6.28

y = hyp sin θ
x = 6.50 sin 345°
x = –1.68

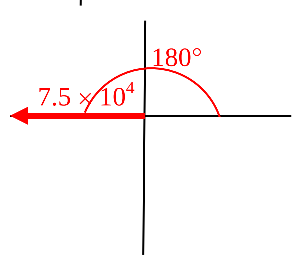
c. 0.00556 at 60°



x = hyp cos θ
x = 0.00556 cos 60°
x = 0.00278

y = hyp sin θ
x = 0.0055 sin 60°
x = 0.0048

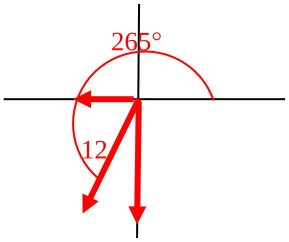
d. 7.5×10^4 at 1



x = hyp cos θ
x = 75,000 cos 180°
x = –75,000

y = hyp sin θ
x = 75,000 sin 180°
x = 0

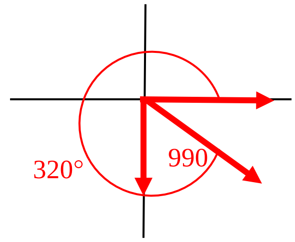
e. 12 at 265°



x = hyp cos θ
x = 12 cos 265°
x = –1.05

y = hyp sin θ
x = 12 sin 265°
x = –11.95

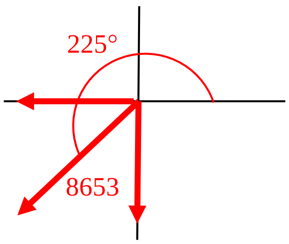
f. 990 at 320°



x = hyp cos θ
x = 990 cos 320°
x = 758.38

y = hyp sin θ
x = 990 sin 320°
x = –636.36

g. 8653 at 225°

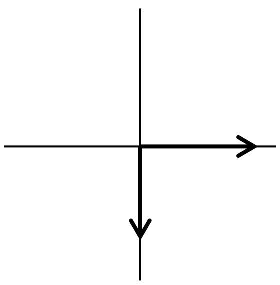


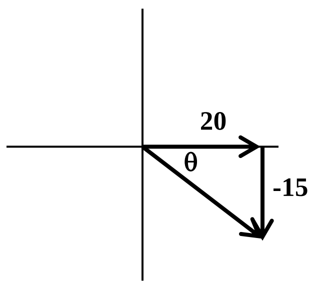
x = hyp cos θ
x = 8653 cos 225°
x = –6118.59

y = hyp sin θ
x = 8653 sin 225°
x = –6118.59

Given two component vectors solve for the resultant vector. This is the opposite of number 10 above. Use Pythagorean Theorem to find the hypotenuse, then use inverse (arc) tangent to solve for the angle.

Example: x = 20, y = –15

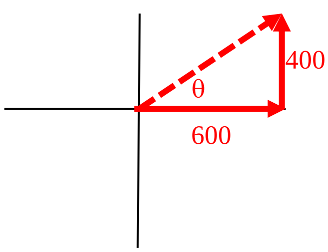




$$R^2 = x^2 + y^2$$
$$R = \sqrt{x^2 + y^2}$$
$$R = \sqrt{20^2 + 15^2}$$
$$R = 25$$
$$\theta = \tan^{-1}\left(\frac{-15}{20}\right) = -36.9^o$$
$$360^o - 36.9^o = 323.1^o$$

$$\tan\theta = \frac{opp}{adj}$$
$$\theta = tan^{-1}\frac{opp.}{adj.}$$
$$\theta = tan^{-1}\frac{y}{x}$$

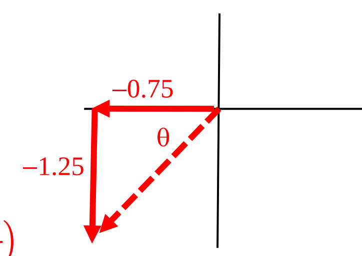
a. x = 600, y = 400



$R^2 = x^2 + y^2$
R = $\sqrt{x^2 + y^2}$
R = $\sqrt{600^2 + 400^2}$
R = 721.11

$\tan\theta = \frac{opp.}{adj.}$
$\rightarrow \tan\theta = \frac{400}{600}$
$\rightarrow tan^{-1}\left(\tan\theta = \frac{400}{600}\right)$
$\rightarrow \theta = tan^{-1}(0.667\ldots)$
$\rightarrow$ **$\theta = 33.70°$**

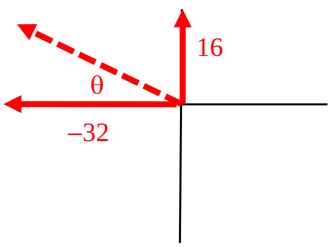
b. x = –0.75, y = –1.25



$R^2 = x^2 + y^2$
R = $\sqrt{x^2 + y^2}$
R = $\sqrt{(-0.75)^2 + (-1.25)^2}$
R = 1.46

$\tan\theta = \frac{opp.}{adj.}$
$\rightarrow \tan\theta = \frac{-1.25}{-0.75}$
$\rightarrow tan^{-1}\left(\tan\theta = \frac{-1.25}{-0.75}\right)$
$\rightarrow \theta = tan^{-1}(1.67)$
$\rightarrow$ **$\theta = 239.04°$**

c. $x = -32,\ y = 16$



$R^2 = x^2 + y^2$

$R = \sqrt{(-32)^2 + (16)^2}$

$R = \sqrt{1024 + 256}$

$R = 35.78$

$\tan\theta = \frac{opp.}{adj.}$

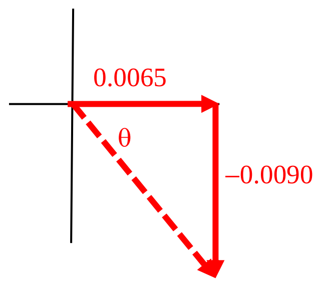
$\rightarrow \tan\theta = \frac{16}{-32}$

$\rightarrow tan^{-1}\left(\tan\theta = \frac{16}{-32}\right)$

$\rightarrow \theta = tan^{-1}(-0.5)$

$\rightarrow$ **$\theta = 153.43°$**

d. $x = 0.0065,\ y = -0.0090$



$R^2 = x^2 + y^2$

$R = \sqrt{x^2 + y^2}$

$R = \sqrt{0.0065^2 + -0.009^2}$

$R = 0.011$

$\tan\theta = \frac{opp.}{adj.}$

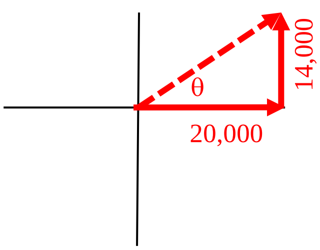
$\rightarrow \tan\theta = \frac{-0.0090}{0.0065}$

$\rightarrow tan^{-1}\left(\tan\theta = \frac{-0.0090}{0.0065}\right)$

$\rightarrow \theta = tan^{-1}(-1.38\ldots)$

$\rightarrow$ **$\theta = 305.84°$**

e. $x = 20{,}000,\ y = 14{,}000$



$R^2 = x^2 + y^2$

$R = \sqrt{x^2 + y^2}$

$R = \sqrt{(20000)^2 + (14000)^2}$

$R = \sqrt{596{,}000{,}000}$

$R = 24{,}413.11$

$\tan\theta = \frac{opp.}{adj.}$

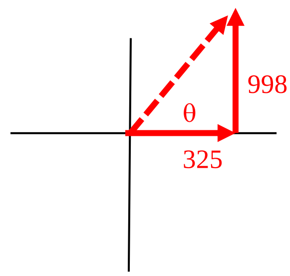
$\rightarrow \tan\theta = \frac{14{,}000}{20{,}000}$

$\rightarrow tan^{-1}\left(\tan\theta = \frac{14{,}000}{20{,}000}\right)$

$\rightarrow \theta = tan^{-1}(0.7)$

$\rightarrow$ **$\theta = 34.99°$**

f. $x = 325,\ y = 998$



$R^2 = x^2 + y^2$

$R = \sqrt{x^2 + y^2}$

$R = \sqrt{325^2 + 998^2}$

$R = \sqrt{1{,}101{,}629}$

$R = 1049.59$

$\tan\theta = \frac{opp.}{adj.}$

$\rightarrow \tan\theta = \frac{998}{325}$

$\rightarrow tan^{-1}\left(\tan\theta = \frac{998}{325}\right)$

$\rightarrow \theta = tan^{-1}(3.07\ldots)$

$\rightarrow$ **$\theta = 71.96°$**

How are vectors used in Physics?

They are used everywhere!

Speed

Speed is a scalar. It only has magnitude (numerical value).

v_s = 10 *m/s* means that an object is going 10 meters every second. But, we do not know where it is going.

Velocity

Velocity is a vector. It is composed of both magnitude and direction. Speed is a part (numerical value) of velocity.

v = 10 *m/s* north, or ***v*** = 10 *m/s* in the **+*x*** direction, etc.

There are three types of speed and three types of velocity

Instantaneous speed / velocity: The speed or velocity at an instant in time. You look down at your speedometer and it says 20 *m/s*. You are traveling at 20 *m/s* at that instant. Your speed or velocity could be changing, but at that moment it is 20 *m/s*.

Average speed / velocity: If you take a trip you might go slow part of the way and fast at other times. If you take the total distance traveled divided by the time traveled you get the average speed over the whole trip. If you looked at your speedometer from time to time you would have recorded a variety of instantaneous speeds. You could go 0 *m/s* in a gas station, or at a light. You could go 30 *m/s* on the highway, and only go 10 *m/s* on surface streets. But, while there are many instantaneous speeds there is only one average speed for the whole trip.

Constant speed / velocity: If you have cruise control you might travel the whole time at one constant speed. If this is the case then you average speed will equal this constant speed.

A trick question

Will an object traveling at a <u>constant speed</u> of 10 *m/s* also always have <u>constant velocity</u>?

Not always. If the object is turning around a curve or moving in a circle it can have a constant speed of 10 *m/s*, but since it is turning, its direction is changing. And if direction is changing then velocity must change, since velocity is made up of speed and direction.

<u>Constant velocity must have both constant magnitude and constant direction.</u>

Rate

Speed and velocity are rates. A rate is a way to quantify anything that takes place during a time interval. Rates are easily recognized. They always have time in the denominator.

10 *m/s* 10 meters / second

The very first Physics Equation

Velocity and Speed both share the same equation. Remember speed is the numerical (magnitude) part of velocity. Velocity only differs from speed in that it specifies a direction.

$v = \frac{x}{t}$ *v* stands for velocity ***x*** stands for displacement ***t*** stands for time

Displacement is a vector for distance traveled in a straight line. It goes with velocity. Distance is a scalar and goes with speed. Displacement is measured from the origin. It is a value of how far away from the origin you are at the end of the problem. The direction of a displacement is the shortest straight line from the location at the beginning of the problem to the location at the end of the problem.

How do distance and displacement differ? Supposes you walk 20 meters down the ***+ x*** axis and turn around and walk 10 meters down the ***– x*** axis.

The distance traveled does not depend on direction since it is a scalar, so you walked 20 + 10 = 30 meter.

Displacement only cares about you distance from the origin at the end of the problem. +20 – 10 = 10 meter.

9. Attempt to solve the following problems. Take heed of the following.

Always use the *KMS* system: Units must be in kilograms, meters, seconds.

On the AP exam you must:

1. List the original equation used.
2. Show correct substitution.
3. Arrive at the correct answer with correct units.

Distance and displacement are measured in meters (*m*)

Speed and velocity are measured in meters per second (*m/s*)

Time is measured in seconds (*s*)

Example: A car travels 1000 meters in 10 seconds. What is its velocity?

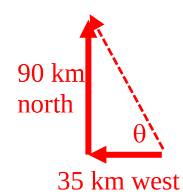
$$v = \frac{x}{t} \qquad v = \frac{1000m}{10s} \qquad v = 100\, m/s$$

a. A car travels 35 *km* west and 75 km east. What distance did it travel?
Ans. The **distance** is the whole path of the car regardless of the direction.
x = 35 km + 75 km = **110 km** = 110 km × $\frac{\mathbf{1000\ m}}{\mathbf{1\ km}}$ = **110,000 m**

b. A car travels 35 *km* west and 75 *km* east. What is its displacement?
Ans. The **displacement** is the distance the car is from the origin. Direction is important.
x = 35 km – 75 km = –45 **km** = –45 **km** × $\frac{\mathbf{1000\ m}}{\mathbf{1\ km}}$ = **–45,000 m**

c. A car travels 35 *km* west, 90 *km* north. What distance did it travel?
Ans. The **distance** is the whole path of the car regardless of the direction.
x = 35 km + 90 km = **125 km** × $\frac{\mathbf{1000\ m}}{\mathbf{1\ km}}$ = **125,000 m**

d. A car travels 35 *km* west, 90 *km* north. What is its displacement?



Ans. The **displacement** is the distance the car is from the origin. Direction is important.
Here we have to find the straight line distance from the beginning to the end of the car's path. This is equivalent to the hypotenuse of the right triangle formed at right
You can use the **Pythagorean Theorem** to find the missing side: $\mathbf{a^2 + b^2 = c^2}$
c = $\sqrt{a^2+b^2} = \sqrt{(35\ km)^2 + (90\ km)^2} = \sqrt{9325\ km}$ = **96.57 km** $\times \frac{1000\ m}{1\ km}$ = **96,570 m**
The **direction** is given by the **angle** that the displacement vector makes with the **horizontal** (see **θ** in the figure above) **.** To find the angle we need to use an **inverse trig function.**

$$\tan\theta = \frac{opp.}{adj.} \rightarrow \tan\theta = \frac{90\ km}{35\ km} \rightarrow tan^{-1}(\tan\theta = \frac{90}{35}) \rightarrow \theta = tan^{-1}(2.571\ldots) \rightarrow \boldsymbol{\theta = 68.75°}$$

e. A bicyclist pedals at 10 *m/s* in 20 *s*. What distance was traveled?
Ans. $v = \frac{x}{t} \rightarrow v \cdot t = x \rightarrow \left(10\frac{m}{s}\right) \cdot (20\ s) = x \rightarrow$ **200 m = x**

f. An airplane flies 250.0 *km* at 300 *m/s*. How long does this take?
Ans. We have "apples" (250 km) and "oranges" (300 m/s) here since they have different units. We need to convert from one unit to the other. $250\ km \times \frac{1000\ m}{1\ km} = 250{,}000\ m$
Ans. $v = \frac{x}{t} \rightarrow v \cdot t = x \rightarrow t = \frac{x}{v} \rightarrow t = \frac{250{,}000\ m}{300\ \frac{m}{s}} =$ **833.3 s**

g. A skydiver falls 3 *km* in 15 *s*. How fast are they going?
Ans. $v = \frac{x}{t} = \frac{3\ km}{15\ s} = 0.6\frac{km}{s} = \frac{0.6\ km}{s} \times \frac{1000\ m}{1\ km} =$ **600 m/s**

h. A car travels 35 *km* west, 90 *km* north in two hours. What is its average speed?

Ans. $v = \frac{x}{t} \rightarrow.\ v = \frac{35\ km + 90\ km}{2\ hr} \rightarrow v = \frac{125\ km}{2\ hr} \rightarrow = \frac{125\ km}{2\ hr} = 62.5 \frac{km}{hr} \times \frac{\mathbf{1000\ m}}{\mathbf{1\ km}} \times \frac{\mathbf{1\ hr}}{\mathbf{3600\ s}}$ = **17.36 m/s**

Average speed is distance over time.

i. A car travels 35 *km* west, 90 *km* north in two hours. What is its average velocity?

Ans. $\vec{v} = \frac{\vec{x}}{t} \rightarrow.\ \vec{v} = \frac{96{,}570\ m}{2\ hr} \rightarrow$ The displacement is the same as **Probl. d.**

$\rightarrow \vec{v} = 48{,}285 \frac{m}{hr} \times \frac{\mathbf{1\ hr}}{\mathbf{3600\ s}}$ = **13.41 m/s**

Average velocity is displacement over time. The **direction** is the same as in Probl. d, given by the **angle** that the displacement vector makes with the **horizontal** (see **θ** in the figure above) **.** To find the angle we need to use an **inverse trig function.**

$$\tan\theta = \frac{opp.}{adj.} \rightarrow \tan\theta = \frac{90\ km}{35\ km} \rightarrow tan^{-1}(\tan\theta = \frac{90}{35}) \rightarrow \theta = tan^{-1}(2.571\ \ldots) \rightarrow \boldsymbol{\theta = 68.75°}$$