Problem: Acceleration (CM-1993)

1. ___**A.**__ In the absence of air friction, an object dropped near the surface of the Earth experiences a constant acceleration of about 9.8 m/s^2. This means that the

(A) speed of the object increases 9.8 m/s during each second
(B) speed of the object as it falls is 9.8 m/s
(C) object falls 9.8 meters during each second
(D) object falls 9.8 meters during the first second only
(E) derivative of the distance with respect to time for the object equals 9.8 m/s^2

Expl. The **acceleration** due to **gravity increases** the **velocity** of a falling object by **9.8 m/s per second.** **Answer B** is wrong because **velocity increases.** **Answer C** is wrong because the **distance increases every second.** **Ans. E** is wrong because the **derivative of distance** with respect to **time** gives the **velocity,** not the **acceleration.**

Problem: Free Fall (B-1993)

2. ___**C.**__ An object is released from rest on a planet that has no atmosphere. The object falls freely for 3.0 meters in the first second. What is the magnitude of the acceleration due to gravity on the planet?

(A) l .5 m/s^2
(B) 3.0 m/s^2
(C) 6.0 m/s^2
(D) 10.0 m/s^2
(E) 12.0 m/s^2

Expl. For objects falling from rest,

$y = -½gt^2 \rightarrow$ Solve for "g"

$-3 = -½(g)(1)^2$

$3 = ½(g)(1)^2 \rightarrow 3 = ½g \rightarrow$ **$6\ m/s^2 = g$**

Problem: Free Fall (CM-1993)

3. ___**D.**__ An object is shot vertically upward into the air with a positive initial velocity. Which of the following correctly describes the velocity and acceleration of the object at its maximum elevation?

	Velocity	Acceleration
(A)	Positive	Positive
(B)	Zero	Zero
(C)	Negative	Negative
(D)	**Zero**	**Negative**
(E)	Positive	Negative

Expl. The velocity is zero **because the object is not moving either upwards or downwards momentarily.** Gravity**, however, is always pulling downwards.**

4. ___**A.**__ A ball is in free fall. Its acceleration is:
   (A) downward during both ascent and descent
   (B) downward during ascent and upward during descent
   (C) upward during ascent and downward during descent
   (D) upward during both ascent and descent
   (E) downward at all times except at the very top, when it is zero

Expl. Gravity **is always pulling downwards towards the center of mass of the planet.**

5. ___**C.**__ A stone is released from a balloon that is descending at a constant speed of 10 m/s. Neglecting air resistance, after 20 s the speed of the stone is:
   (A) 2160 m/s (D) 196 m/s
   (B) 1760 m/s (E) 186 m/s
   (C) 210 m/s

Expl. The speed of the balloon is given by $v = v_o + at = -10 - 10(20) = -210$ m/s
Since speed is directionless, the answer is |–210 m/s|.

6. ___**A.**__ An object is thrown vertically upward with a certain initial velocity in a world where the acceleration due to gravity is 19.6 m/s^2. The height to which it rises is ____ that to which the object would rise if thrown upward with the same initial velocity on the Earth. Neglect friction.
   (A) half (D) four times
   (B) $\sqrt{2}$ times (E) cannot be calculated
   (C) twice

Expl. We can throw out any answers that are greater than one (Ans. C and D) since gravity will pull down an object faster on this world since g is $2g_{earth}$.

Let's try a value and see what happens. Assume $v_o = 9.8$ m/s. At the top of its flight the velocity is 0 m/s so we can find the time it takes for an object to be in the air on Planet Earth where $g = 9.8\ m/s^2$:

$$\mathbf{v = v_o - gt}$$
$$0 = 9.8\ m/s - 9.8t$$
$$t = 1\ sec$$

On a planet with twice the gravity, 2g:

$$\mathbf{v = v_o - 2gt}$$
$$0 = 9.8\ m/s - 19.6t$$
$$t = 1/2\ sec$$

Maximum height		Differ.
On other world	$y = (9.8\ m/s)(0.5) - \frac{1}{2}\,\mathbf{19.6(0.5)^2}$ $y = 2.45$ m	**2.45 m**
On Earth	$y = (9.8\ m/s)(1) - \frac{1}{2}\mathbf{9.8(1)^2}$ $y = 4.9$ m	

7. ___**B.**__ A ball is released from rest near the surface of the Moon. Which **one** of the following quantities increases at a constant rate?
   (A) Only distance fallen
   (B) Only speed
   (C) Only speed and distance fallen
   (D) Only speed and acceleration

Expl. Only speed because the ball is subject to the *constant* acceleration due to gravity. Acceleration does not increase, it stays constant and distance increases according to t^2, getting bigger every second.

FREE RESPONSE

1. You are on top of a building 45 meters high when you toss a ball **upward** with an initial velocity of 52.0 m/s. This ball then lands on the roof of an adjacent building which is 155 meters high.

a) How long will it take for the ball to reach the adjacent roof?

Ans. The **general formula** for **vertical height** is $y(t) = y_o + v_{yo}t - ½gt^2$.
Substituting our values: $155 \text{ m} = 45 \text{ m} + (52 \text{ m/s})t - ½(9.8 \text{ m/s}^2)t^2$
Solving for t: $0 = -110 + 52t - 4.9t^2$
$0 = 110 - 52t + 4.9t^2$

Solving on the graphing calculator (x-intercept) or other means: **t = 2.9 sec** and **t = 7.7 sec** are the x- intercepts. We can assume the answer is the second one (**7.7 sec**) because the ball has to go up past the roof, reach its highest point, and then fall back down.

b) What will be the velocity of the ball just as it lands on the roof?

Ans. The formula for **velocity**, $\mathbf{v(t) = v_{yo} - gt = 52 - 9.8t = 52 - 9.8(7.7\text{ sec}) = -23.46\text{ m/s}}$

c) What will be the average velocity of the ball from the time it leaves your hand until it lands on the roof?

Ans. **Average velocity** is $v = \frac{\Delta y}{\Delta t} = \frac{45+52(7.7)-4.9(7.7)^2-(45+52(0)-4.9(0)^2)}{7.7-0} = \frac{109.879}{7.7} =$ **14.3 m/s**

d) When will the ball be 230 meters above the ground?

Substituting our values: $230 \text{ m} = 45 \text{ m} + (52 \text{ m/s})t - ½(9.8 \text{ m/s}^2)t^2$
Solving for t: $0 = -185 + 52t - 4.9t^2$
$0 = 185 - 52t + 4.9t^2$

Solving on the graphing calculator (x-intercept) or other means: **at no time t.** The **maximum height** of the ball is **182.95 m**.

e) What will be the speed of the ball when it is 230 meters above the ground?

Ans. Not applicable. The ball doesn't reach this height.

f) What will be the highest point reached by this ball?

Ans. The time to reach the highest point will happen when $v_y = 0$.

$v(t) = 52 - 9.8t \rightarrow$

$0 = 52 - 9.8t$

$-52 = -9.8t \rightarrow 5.3 = t$

At t = 5.3 s, $y = 45\text{ m} + 52(\mathbf{5.3\ s}) - ½(9.8\text{ m/s}^2)(\mathbf{5.3\ s})^2 =$ **182.95 m**

g) What will be the velocity of this ball at the highest point?

Ans. At the highest point the velocity is **0 m/s**. The ball is not moving up or down.

2. A hot air balloon is initially sitting on the roof of a building which is 120 meters tall. At t = 0 sec the balloon is released and initially accelerates upward at 1.2 m/sec^2 until after 5.0 seconds the balloon finally achieves its maximum upward speed. Exactly 4.0 seconds after the balloon achieves its maximum speed a brick accidentally is dropped from the balloon.

a) What was the maximum velocity of the balloon?

Ans. The hot air balloon rises according to $v_y = at = (\mathbf{1.2\ m/sec^2})(\mathbf{5.0\ s}) =$ **6 m/s**

b) What was the height of the balloon when it reached the maximum velocity?

Ans. The hot air balloon rises according to $y = ½at^2 = ½(\mathbf{1.2\ m/sec^2})(\mathbf{5.0\ s})^2 =$ **15 m**

c) What was the height of the balloon when the brick was dropped?

Ans. The balloon continues upwards at a **constant velocity** at its **maximum velocity** of **6 m/s**. After 4 seconds, its height would be **$y = vt$** $= 6\text{ m/s}\cdot(4\text{ s}) =$ **24 m**. Its **total height** would thus be 120 m + 15 m + 6 m = **141 m**

d) What was the velocity of the brick as it was dropped from the balloon?

Ans. The **initial velocity** of the brick would be the same as the **velocity** of the balloon, **+6 m/s**.

e) What was the maximum height reached by the brick?

Ans. The **height formula** for the brick would be $\mathbf{y(t) = y_o + v_{yo}t - ½gt^2}$

$\mathbf{y(t)} = \mathbf{141} + 6t - ½(9.8)t^2$

$\mathbf{y(t)} = \mathbf{141} + 6t - 4.9t^2$

At the highest point, the **velocity** of the brick will be **zero**. This happens at:

$v = 6 - 9.8t \rightarrow 0 = 6 - 9.8t \rightarrow -6 = -9.8t \rightarrow -6 = -9.8t \rightarrow 0.61\text{ s} = t$

The height of the brick would be: $\mathbf{y(t)} = 141 + 6t - 4.9t^2 = 141 + 6(\mathbf{0.61\ s}) - 4.9(\mathbf{0.61\ s})^2 =$ **142.84 m**

f) How long after it was dropped did the brick strike the ground?

Ans. When will the brick have a height of 0 m? Set the height equation = 0.

$\mathbf{y(t)} = \mathbf{141} + 6t - 4.9t^2 \rightarrow$

$0 = 141 + 6t - 4.9t^2$

Using a **graphing calculator** to solve for the x-intercept: **t = 6.01 s**