PHYSICS HOMEWORK #41 ENERGY CONSERVATION
WORK & ENERGY

ANSWER KEY

WORK & ENERGY

1. A force of 25.0 Newtons is applied so as to move a 5.0 kg mass a distance of 20.0 meters. How much work was done?
   Ans. W = F × d = 25 N × 20.0 m **= 50 J**
2. A force of 120 N is applied to the front of a sled at an angle of 28.0° above the horizontal so as to pull the sled a distance of 165 meters. How much work was done by the applied force?
   Ans. W = F •d cos θ = **120 N • 165 m** cos **28.0°** = **17,482.36 J**

3. A sled, which has a mass of 45.0 kg., is sitting on a horizontal surface. A force of 120 N is applied to a rope attached to the front of the sled such that the angle between the front of the sled and the horizontal is 35.0º. As a result of the application of this force the sled is pulled a distance of 500 meters at a relatively constant speed. How much work was done to this sled by the applied force?
   Ans. W = F• d cos θ = 120 N • 500 m • cos **35.0°** = **49,149.12 N**

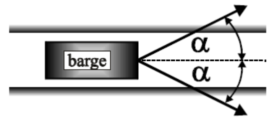
4. A rubber stopper, which has a mass of 38.0 grams, is being swung in a horizontal circle which has a radius of R = 1.35 meters. The rubber stopper is measured to complete 10 revolutions in 8.25 seconds.

   a. What is the speed of the rubber stopper?
   Ans. $\mathbf{v = \frac{d}{t} \rightarrow v = \frac{n \bullet 2\pi r}{t} \rightarrow v = \frac{10\ rev. \bullet 2\pi(1.35\ m)}{8.25\ s} \rightarrow v =}$ **10.29 m/s**
   b. How much force must be applied to the string in order to keep this stopper moving in this circular path at a constant speed?
   Ans. The **Centripetal Force,** $\mathbf{F_c = m \bullet \frac{v^2}{r} = 0.38\ kg \bullet \frac{(10.29\ m/s)^2}{1.35\ m} =}$ **29.76 N**
   c. How far will the stopper move during a period of 25.0 seconds?
   Ans. d = v·t = 10.29 m/s·25 s = 257.25 m

   d. How much work is done on the stopper by the force applied by the string during 25.0 seconds?
   Ans. 0 J. The force is towards the center and the distance is around the perimeter of the circle. They are at right angles to each other. For work to be done, the force has to be in the same direction as the displacement.

5. How much work would be required to lift a 12.0 kg mass up onto a table 1.15 meters high?
   Ans. W = F • d = mg d = 12 kg · 9.8 m/s^2 · 1.15 m = **135.24 J**

6. A barge is being pulled along a canal by two cables being pulled as shown to the right. The tension in each cable is T =14,000 N and each cable is being pulled at an angle α = 18.0° relative to the direction of motion as shown. How much work will be done in pulling this barge a distance of 3.0 kilometers?



Ans. The **component** of the **force** along the **direction** of motion is $\mathbf{F_x = T \cdot \cos\theta}$

$\mathbf{F_x}$ **=** 14,000 N cos 18° **= 13,314.79 N**

Since there are two such cables, the **total force** along the **x-direction** is

$\mathbf{F_x}$ **=** 13,314.79 N × **2 = 26,629.58 N**

W = F × d = **26,629.58 N × 3000 m = 79,888,747.37 J = 79 MegaJoules**

Kinetic Energy KE = ½ mv^2

7. A car, which has a mass of 1250 kg is moving with a velocity of 26.0 m/sec. What is the kinetic energy of this car?

Ans. TKE = ½mv^2 = ½(1250 kg)(26 m/s)^2 = **422,500 J**

8. What will be the kinetic energy of a bullet, which has a mass of 22.0 grams, moving with a velocity of 650 m/sec.?

Ans. TKE = ½mv^2 = ½(0.022 kg)(650 m/s)^2 = **4647.5 J**

9. How fast must a 4.40 kg bowling ball move in order to have a kinetic energy of 185 Joules?

Ans. TKE = ½mv^2 → 185 J = ½(4.40 kg)v^2 → 185 = 2.20v^2 → 84.09 = v^2 → √84.09 = √v^2

9.17 m/s = v

10. A ball, which has a mass of 2.40 kg., is dropped from the top of a building 96.0 meters tall.

a. How long will it take for this ball to reach the ground?

Ans. $\mathbf{y = \frac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2y}{g}} \rightarrow t = \sqrt{\frac{2(96\ m)}{9.8\frac{m}{s^2}}} = 4.42\ sec}$

b. What will be the velocity of the ball just as it reaches the ground?

Ans. $\mathbf{v = -gt = 9.8\frac{m}{s^2} \cdot 4.42\ sec = 43.38\ m/s}$

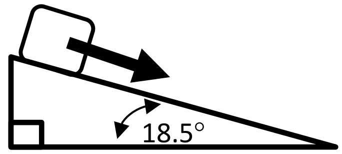
c. What will be the kinetic energy of the ball just as it reaches the ground?

Ans. TKE = ½mv^2 = ½(2.40 kg)(43.38 m/s)^2 = **2258.19 J**

d. How much work would be needed to lift this ball back up to the top of the building at a constant speed?

Ans. 2258.19 J. Energy is conserved. The amount of **kinetic energy** is the same as the amount of **work** to reset the ball to its **original height**.

11. A cart, which has a mass of m = 2.50 kg., is sitting at the top of an inclined plane which is 3.30 meters long and which meets the horizontal at an angle of $\beta = 18.5°$.



a. How long will it take for this cart to reach the bottom of the inclined plane?

Ans. The **acceleration** of a cart down an inclined plane is given by **a = g sin θ, where g is gravity** and θ is the **angle** of the **inclined plane.**

$a = \mathbf{9.8\frac{m}{s^2}\cdot \sin 18.5° = 3.11\ m/s^2}$

The **distance** an object moves down an **inclined plane** when it is **accelerating** is given by $\mathbf{x = ½at^2}$ **Solving for t:** $\mathbf{t = \sqrt{\frac{2x}{a}} \rightarrow t = \sqrt{\frac{2(3.30\ m)}{3.11\frac{m}{s^2}}} = 1.46\ sec}$

b. What will be the velocity of the cart when it reaches the bottom of the incline?

Ans. The **velocity** of an object at the bottom of an **inclined plane** is

$\mathbf{v = a\cdot t = 3.11\ m/s^2\cdot 1.46\ s = 4.53\ m/s}$

c. What will be the kinetic energy of the cart when it reaches the bottom of the incline?

Ans. $TKE = ½mv^2 = ½(2.50\ kg)(4.53\ m/s)^2 = \mathbf{25.66\ J}$

Gravitational Potential Energy GPE = mgΔh

12. A 5.0 kg mass is initially sitting on the floor when it is lifted onto a table 1.15 meters high at a constant speed.

a. How much work will be done in lifting this mass onto the table?

Ans. $\mathbf{W = F \times d = m\cdot g \times d = 5.0\ kg \cdot 9.8\frac{m}{s^2}\cdot 1.15\ m = 56.35\ J}$

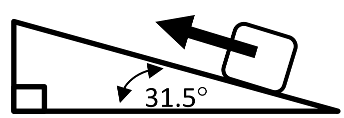
b. What will be the gravitational potential energy of this mass, relative to the floor, once it is placed on the table?

Ans. 56.35 J. By **Conservation of Energy,** the **Work** had to be converted into another form of **energy.** In this case, it is **gravitational potential energy.**

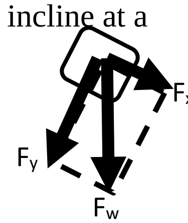
c. What was the initial gravitational potential energy, relative to the floor, of this mass while sitting on the floor?

Ans. 0 J. If the mass had **no height,** it had **no GPE.**

13. A crate, which has a mass of 48.0 kg., is sitting at rest at the bottom of a frictionless inclined plane which is L = 2.85 meters long and which meets the horizontal at an angle of $\alpha = 31.5°$. A force F is applied so as to push the crate up this incline at a constant speed.



a. What is the magnitude of the force F required to push the crate to the top of the incline at a constant speed?

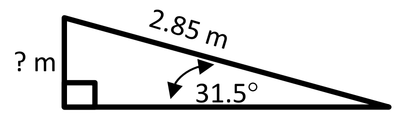


Ans. To get the crate to go **up** the **inclined plane,** you would have to apply a **force equal** and **opposite to F_x, the component** of the **crate's weight** ***along the inclined plane.***

$\mathbf{F_x = F_w\cdot \sin\theta} = \mathbf{m\cdot g}\cdot \sin\theta = \mathbf{48\ kg\cdot 9.8\frac{m}{s^2}}\cdot \sin \mathbf{31.5°} = \mathbf{245.78\ N}$

b. How much work will be done in pushing the crate to the top of the incline?

Ans. W = F × d = 245.78 N × 2.85 m = 700.48 J



c. What is the height of this incline?

Ans. We have the **hypotenuse** and the **angle θ** and we are trying to find the **opposite side.**

We should use **sin θ** $= \frac{\text{opp.}}{\text{hyp.}} \rightarrow$ **hyp· sin θ = opp.** $\rightarrow$ **2.85 m ·sin (31.5°) = opp.**

1.48 m = opp.

d. What will be the GPE of this crate when it reaches the top of the incline?

Ans. GPE = 700.48 J. By Conservation of energy, Ans. b. is the same as **Ans. d. But to verify, we can find the GPE using our formula.**

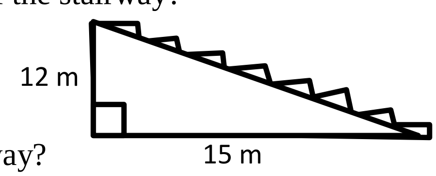
GPE = mgh = 48.0 kg $\cdot\, 9.8\,\frac{\text{m}}{\text{s}^2} \cdot$ **1.48 m = 700.48 J**

14. Suppose that you have a mass of 62.0 kg and that you walk to the top of a stairway which is h = 12.0 meters high and L= 15.0 meters deep.

a. How much work will you have to do in walking to the top of the stairway?

Ans. To get to the **top of the stairs,**

W = F × d=mgh = 62.0 kg $.\, 9.8\,\frac{\text{m}}{\text{s}^2} \cdot$ **12 m =7291.2 J**



b. What will be your GPE when you reach the top of the stairway?

Ans. 7291.2 J. By **Conservation of Energy,** the **work** you did going up the stairs will give you that much **GPE.**

Elastic Potential Energy EPE = ½ k(Δx)^2 and [F = kΔx]

15. A force of F = 35.0 N is applied to a spring and as a result the spring stretches a distance of Δx = 12.0 cm.

a. What is the spring constant for this spring?

Ans. The formula for the **spring constant** is **k** $= \frac{F}{x} \rightarrow$ **k** $= \frac{35\text{ N}}{0.12\text{ m}} =$ **291.67 N/m**

b. How much energy will be stored in this spring?

Ans. The energy of a spring is **elastic potential energy.**

EPE = ½kx^2 = ½(291.67 N/m)(0.12 m)^2 = 2.1 J

16. A spring, which has a spring constant of k = 120 N/m, is being stretched a distance of Δx = 15.0 cm by a force F.

a. How much force F is being applied to this spring?

Ans. Hooke's law (which is the formula for the **spring constant** is **F = kx)**

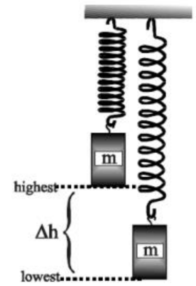
F = k·x = 120 N/m· 0.15 m = 18 N

b. How much energy will be stored in this spring?

Ans. The energy of a spring is **elastic potential energy.**

EPE = ½kx^2 = ½(120 N/m)(0.15 m)^2 = 1.35 J

17. A spring, which has a spring constant k, is hung from the ceiling as shown to the right. A mass m = 3.00 kg is added to the end of the spring and is then slowly lowered until equilibrium is reached. At this point the bottom of the mass has been lowered a distance of Δh = 52.0 cm.



a. What is the magnitude of the force being exerted by the spring when the system reaches equilibrium?

Ans. The **spring** must be exerting **a force equal** and **opposite** to the **weight** of the **hanging mass** for **equilibrium** to occur.

$F_W = m \cdot g = 3\text{ kg} \cdot 9.8 \frac{m}{s^2} = \mathbf{29.4\ N}$

b. What is the spring constant of this spring?

Ans. The formula for the **spring constant** is $k = \frac{F}{x} \rightarrow k = \frac{29.4\text{ N}}{0.52\text{ m}} = \mathbf{56.54\ N/m}$

c. How much energy is stored in the spring when equilibrium is reached?

Ans. The energy of a spring is **elastic potential energy.**

$EPE = \frac{1}{2}kx^2 = \frac{1}{2}(56.54\text{ N/m})(0.52\text{ m})^2 = \mathbf{7.64\ J}$

18. A mass of 5.00 kg is dropped from a height of 2.20 meters above a vertical spring sitting on a horizontal surface. Upon colliding with the spring the mass compresses the spring Δx = 30.0 cm before it momentarily comes to halt. [Assume h = 0 at the lowest point!]

a. How much gravitational potential energy was contained in the 5.0 kg mass before it was dropped?

Ans. $GPE = mgh = 5.00\text{ kg} \cdot 9.8 \frac{m}{s^2} \cdot 2.20 + 0.30\text{ m} = \mathbf{122.5\ J}$

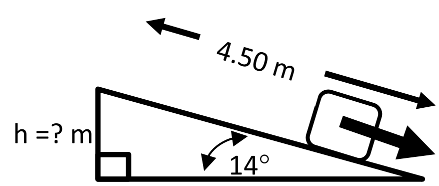
b. How much energy will be stored in the spring when the mass comes briefly to a halt?

Ans. **All** the **GPE** will **convert** to **EPE.** **122.5 J**

c. What is the spring constant of this spring?

Ans. The formula for the **spring constant** is $k = \frac{F}{x} \rightarrow k = \frac{m \cdot g}{x} \rightarrow k = \frac{5\text{ kg} \cdot 9.8\frac{m}{s^2}}{2.50\text{ m}} = \mathbf{19.6\ N/m}$

ENERGY CONSERVATION:



19. A cart, which has a mass of 2.30 kg is sitting at the top of an inclined plane, which is 4.50 meters long and meets the horizontal at an angle of 14.0º. The car is then allowed to roll to the bottom of the incline;

a. What was the gravitational energy of the cart before it rolls down the incline?

Ans. GPE = mgh. We know the **mass** is 2.30 kg and we know **gravity** is $9.8 \frac{m}{s^2}$. We need ***h***.

We have the **hypotenuse** and the **angle θ** and we are trying to find the **opposite side, *h*.**

We should use $\sin\theta = \frac{opp.}{hyp.} \rightarrow hyp \cdot \sin\theta = opp. \rightarrow 4.50\text{ m} \cdot \sin(14°) = opp.$

$1.09\text{ m} = opp. = h$

$GPE = 2.30\text{ kg} \cdot 9.8 \frac{m}{s^2} \cdot 4.50\text{ m} = \mathbf{101.43\ J}$

b. What will be the magnitude of the force that tends to cause the cart to accelerate down the incline?

Ans. F_x, the component of the **crate's weight *along the inclined plane,*** is what makes the cart accelerate **down** the plane:

F_x = F_w·sin θ = **m·g**·sin θ = **2.30 kg**· $\mathbf{9.8\frac{m}{s^2}}$ · sin **14°** = **5.45 N**

c. What will be the acceleration of the cart as it moves down the incline?

Ans. The **acceleration** of a cart down an inclined plane is given by **a = g sin θ, where g is gravity** and **θ** is the **angle** of the **inclined plane.**

a = $\mathbf{9.8\frac{m}{s^2}}$· **sin 14° = 2.37 m/s²**

d. How much time to it take for the cart to reach the bottom of the incline?

Ans. The **distance** an object moves down an **inclined plane** when it is **accelerating** is given by **$x = \frac{1}{2}at^2$ Solving for t:** $\mathbf{t = \sqrt{\frac{2x}{a}} \rightarrow t = \sqrt{\frac{2(4.50\ m)}{2.37\frac{m}{s^2}}}}$ = **1.95 sec**

e. What will be the velocity of the cart as it reaches the bottom of the incline?

Ans. The **velocity** of an object at the bottom of an **inclined plane** is

v = a·t = 2.37 m/s²·1.95 s = 4.62 m/s

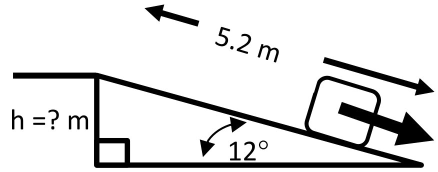
f. What will be the kinetic energy of the cart as it reaches the bottom of the incline?

Ans. TKE = ½mv² = ½(**2.30 kg**)(**4.62 m/s**)² = **24.55 J**

g. How much work was done by the gravitational force on the cart as it rolls to the bottom of the incline?

Ans. W = 24.55 J. The **gravitational force,** by **Conservation of Energy,** must have been equal to the **Kinetic Energy** gained.

20. A car is sitting at the top of an inclined plane, which is 5.2 meters long and meets the horizontal at an angle of 12.0º. The cart is then allowed to roll to the bottom of the incline. What will be the velocity of the cart as it reaches the bottom of the incline?



Ans. To find the **velocity *at the bottom,*** we can use **v =** $\sqrt{2gh}$ but we need the height. We need ***h***.

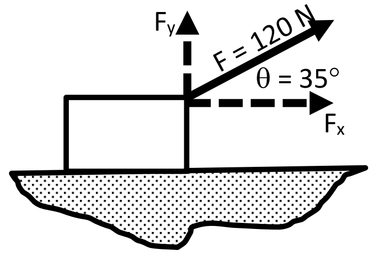
We have the **hypotenuse** and the **angle θ** and we are trying to find the **opposite side, *h*.**

We should use **sin θ** = $\frac{\text{opp.}}{\text{hyp.}}$ → **hyp· sin θ = opp.** → **5.20 m ·sin (12°) = opp.**

1.08 m = opp. = h

v = $\sqrt{2gh}$ = $\sqrt{2\,(9.8\frac{m}{s^2})(1.08\,m)}$ = **4.6 m/s**

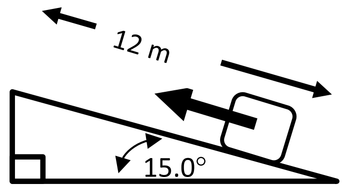
21. A sled, which has a mass of 45.0 kg., is sitting on a horizontal surface. A force of 120 N is applied to a rope attached to the front of the sled such that the angle between the front of the sled and the horizontal is 35.0º. As a result of the application of this force the sled is pulled a distance of 500 meters at a relatively constant speed. How much work was done to this sled by the applied force?



Ans. $\mathbf{W = F \times d = F_x \times d = F \cos\theta \cdot d = 120\ N \cos 35° \cdot 500\ m =}$ $\mathbf{49{,}149.12\ J}$

We are only interested in the component of the Force *along the horizontal surface*. This would be F_x (see figure at right).

22. A 25.0 kg crate is sitting at the bottom of an inclined plane. The inclined plane is 12.0 meters long, meets the horizontal at an angle of 15.0º and has a coefficient of sliding friction of μ = 0.55. A force is applied to the crate so as to slide the crate up the incline at a constant speed.

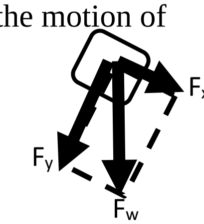


a. What will be the magnitude of the frictional force between the crate and the incline?

Ans. The formula for the **frictional force *on an inclined plane*** is $\mathbf{F_f = \mu F_N = \mu F_y = \mu \cdot mg \cdot \cos\theta}$

$\mathbf{F_f = 0.55 \cdot 25.0\ kg\left(9.8\frac{m}{s^2}\right)\cos 15° \ = 130.16\ N}$

b. What will be the magnitude of the gravitational force component opposing the motion of the sled up the incline?



Ans. The component of the weight of the crate ***along the inclined plane*** is just the **opposite side** of the **right triangle** formed by F_x, F_y, and F_w (see figure at right).

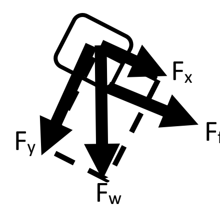
$\mathbf{F_x = m \cdot g \sin\theta \ = 25.0\ kg\left(9.8\frac{m}{s^2}\right)\sin 15° = 63.41\ N}$

c. How much work will be done against the gravitational force in moving the crate to the top of the incline?

Ans. The **work** required against the **gravitational force *alone*** is $\mathbf{W = F \times d = -F_x \times d}$

$\mathbf{W = 63.41\ N \times (12\ m) = -760.93\ J}$

d. What is the magnitude of the force F required to push the sled up the incline at a constant speed?



Ans. Since the crate is moving up the plane **at a constant speed,** the forces **up the plane** must be equal to the **forces down the plane.**

$\mathbf{F\uparrow = F\downarrow \ \rightarrow F_p = F_x + F_f = 63.41\ N + 130.16\ N}$
$\mathbf{= 193.57\ N}$

The **forces down** are the **force of friction (F_f)** but also the ***component of the weight*** of the crate that is ***along the inclined plane*** **(F_x).**

e. How much work will be done by the applied force in pushing the mass to the top of the incline?

Ans. The **work** required against the **gravitational force (F_x)** ***AND*** **the frictional force (F_f)** is **W = F × d = 193.57 N × (12 m) = 2322.84 J**

f. What will be the gravitational potential energy of the crate when it reaches the top of the incline?

Ans. GPE = mgh. We know the **mass** is 25 kg and we know **gravity** is $\mathbf{9.8\frac{m}{s^2}}$. We need ***h***.

We have the **hypotenuse** and the **angle θ** and we are trying to find the **opposite side, *h*.**

We should use **sin θ** $= \frac{\textbf{opp.}}{\textbf{hyp.}} \rightarrow$ **hyp· sin θ = opp.** → **12 m ·sin (15°) = opp.**

3.11 m = opp. = h

GPE = 25 kg· $\mathbf{9.8\frac{m}{s^2}}$ **· 3.11 m = 760.93 J**

g. How much work was done against the frictional force as the crate is pushed to the top of the incline?

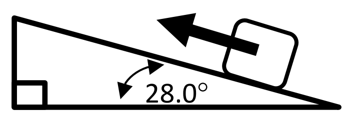
Ans. The **work** required against the **frictional force *alone*** is **W = F × d = –F_f × d**

W = 130.16 N × (12 m) = –1561.92.93 J

h. How are the work done by the external force, the work done against friction and the work done against gravity related?

Ans. $\mathbf{Work_{ext} = W_{frict} + W_{grav}}$

23. A box, which has a mass of 14.0 kg , is sliding along a horizontal surface with a velocity of 18.0 m/sec when it encounters a frictionless inclined plane which meets the horizontal at an angle of 28.0°. The box slides up the incline until it comes to a halt.



a. What will be the kinetic energy of the crate before it reaches the bottom of the incline?

Ans. $\mathbf{TKE = \frac{1}{2}mv^2} = \frac{1}{2}(\mathbf{14\ kg})(\mathbf{18\ m/s})^2 = \mathbf{2268\ J}$

b. What will be the gravitational potential energy of the crate when it finally stops on the incline?

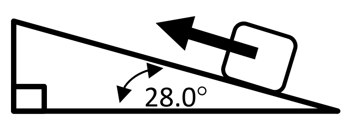
Ans. By **Conservation of energy**, the $\mathbf{GPE_{out} = TKE_{in}} = \mathbf{2268\ J}$

c. How far up the incline will the box slide before it stops?

Ans. **GPE = mgh** → **h =** $\frac{\mathbf{GPE}}{\boldsymbol{mg}}$ → **h =** $\frac{\mathbf{2268\ J}}{14\ kg \cdot 9.8\frac{m}{s^2}}$ **= 16.53 m.**

This is the **height** to which the **box** will rise.

h =16.53 m



To find ***how far up,*** we need to use **S.O.H.C.A.H.T.O.A.**

and in this case **sin θ** = $\frac{\boldsymbol{opp.}}{\boldsymbol{hyp.}}$ → **hyp.** = $\frac{\boldsymbol{opp.}}{\boldsymbol{sin\ \theta}}$ →

hyp. = $\frac{\mathbf{16.53}\ \boldsymbol{m}}{\boldsymbol{sin}\ \mathbf{28}}$ = **35.21 m**