Physics C — Rotational Motion — AP Review Packet — Name: __ANSWER KEY_

Linear and angular analogs

Linear	**Rotation**	
x	θ	position
Δx	$\Delta\theta$	displacement
v	ω	velocity
a_T	α	tangential acceleration

Vectors in rotational motion

Use the right hand rule to determine direction of the vector!

Don't forget centripetal acceleration!

$a_R = a_c = v^2/r$

Kinematic equations for angular and linear motion.

Kinematic Equations 1

$v = v_o + at$

$\omega = \omega_o + \alpha t$

Kinematic Equations 2

$x = x_o + v_o t + {}^1/_2 at^2$

$\theta = \theta_o + \omega_o t + {}^1/_2 \alpha t^2$

Kinematic Equations 3

$v^2 = v_o^2 + 2a(x - x_o)$

$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$

Rotational Inertia

Rotational analog of mass

For point masses

$I = \Sigma mr^2$

I: rotational inertia ($kg\ m^2$)

m: mass (kg)

r: radius of rotation (m)

For solid objects

$I = \int r^2\, dm$

Parallel Axis Theorem

$I = I_{cm} + M h^2$

I: rotational inertia about center of mass

M: mass

h: distance between axis in question and axis through center of mass

Kinetic Energy

$K_{trans} = ½ M v_{cm}^2$

$K_{rot} = ½ I \omega^2$

$K_{combined} = ½ M v_{cm}^2 + ½ I \omega^2$

Rolling without slipping uses both kinds

$K = ½ M v_{cm}^2 + ½ I \omega^2$

$v = \omega r$

$K = ½ M v_{cm}^2 + ½ I_{cm} v_{cm}^2/R^2$

or

$K = ½ M \omega^2 R^2 + ½ I_{cm} \omega^2$

Torque

Torque is the rotational analog of force.

A "twist" (whereas force is a push or pull).

Torque is a vector)

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$\tau = r F \sin\theta$

R: moment arm length

F: force

θ: angle between moment arm and point of application of force.

$\Sigma\boldsymbol{\tau} = I \boldsymbol{\alpha}$ *(think $\Sigma \mathbf{F} = m\mathbf{a}$)*

$\boldsymbol{\tau}$: torque

I: rotational inertia

$\boldsymbol{\alpha}$: angular acceleration

Work in rotating systems

$W_{rot} = \boldsymbol{\tau} \bullet \Delta \boldsymbol{\theta}$ *(think $W = \mathbf{F} \bullet \mathbf{d}$)*

W_{rot} **:** work done in rotation

$\boldsymbol{\tau}$: torque

$\Delta\boldsymbol{\theta}$: angular displacement

Power in rotating systems

$P_{rot} = \boldsymbol{\tau} \bullet \boldsymbol{\omega}$ *(think $P = \mathbf{F} \bullet \mathbf{v}$)*

P_{rot} **:** power expended

$\boldsymbol{\tau}$: torque

$\boldsymbol{\omega}$: angular velocity

Static Equilibrium

$\Sigma\tau = 0$ $\Sigma F = 0$

Angular momentum

For a particle

$\mathbf{L} = \mathbf{r} \times \mathbf{p}$

For a system of particles

$\mathbf{L} = \Sigma\, \mathbf{L_i}$

For a rigid body

$\mathbf{L} = I\, \boldsymbol{\omega}$ *(think $\mathbf{P} = m\mathbf{v}$)*

Conservation of Angular Momentum

Angular momentum of a system will not change unless an external torque is applied to the system.

$L_B = L_A$

$I\omega_B = I\omega_A$ (one body)

$\Sigma\, l_b = \Sigma\, l_a$ (system of particles)

Angular momentum and torque

$\boldsymbol{\tau} = d\mathbf{L}/dt$ *(think $\mathbf{F} = d\mathbf{P}/dt$)*

$\boldsymbol{\tau}$: torque

$\mathbf{L}$: angular momentum

t: time

Torque increases angular momentum when parallel.

Torque decreases angular momentum when antiparallel.

Torque changes the direction of the angular momentum vector in all other situations.

Precession

The rotating motion made by a spinning top or gyroscope. Precession is caused by the interaction of torque and angular momentum vectors.

$\boldsymbol{\tau} = d\mathbf{L} / dt$

$\tau = r \times F$

MULTIPLE CHOICE PRACTICE PROBLEMS

1. ___**D.**__ A wheel spinning at 3 m/s uniformly accelerates to 6 m/s in 4 s. Its radius is 20 cm. How far around the wheel will a speck of dust travel during that interval?

A) 6 m
B) 9 m
C) 12 m
D) 18 m
E) 30 m

Ans. $a = \frac{v - v_o}{t} = \frac{6\frac{m}{s} - 3\frac{m}{s}}{4\ s} = \frac{3}{4}\ m/s^2$

$v^2 = v_o^2 + 2a\Delta x$

$(6\ m/s)^2 = (3\ m/s)^2 + 2(\frac{3}{4}\ m/s^2)x$

$36 = 9 + 3/2x$

$27 = 3/2x$

$18\ m = x$

The radius is not relevant.

2. ___**B.**___ If an object of radius 3 m that experiences a constant angular acceleration starting from rest, rotates 10 rads in 2 s, what is its angular acceleration?

A) 2.5 rad/s²
B) 5 rad/s²
C) 7.5 rad/s²
D) 10 rad/s²
E) 15 rad/s²

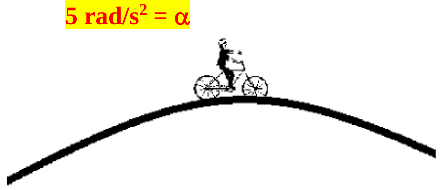
Ans. $\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$

$10\ rad = 0\ rad + (0\ rad/s)(2\ s) + \frac{1}{2}\alpha(2\ s)^2$

$10 = \frac{1}{2}\alpha(2\ s)^2$

$10 = 2\alpha$

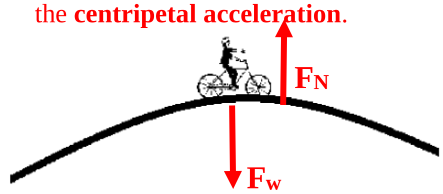
$\mathbf{5\ rad/s^2 = \alpha}$



3. ___**B.**___ A bicycle moves at constant speed over a hill along a smoothly curved surface as shown above. Which of the following best describes the directions of the velocity and the acceleration at the instant it is at the highest position?

A) The velocity is towards the right of the page and the acceleration is towards the top of the page.
B) The velocity is towards the right of the page and the acceleration is towards the bottom of the page.
C) The velocity is towards the right of the page and the acceleration is towards the bottom right of the page.
D) The velocity is towards the right of the page and the acceleration is towards the top right of the page.
E) The velocity is towards the top right of the page and the acceleration is towards the bottom right of the page.

Ans. Since the bike is moving at **constant speed**, we don't have to worry about **tangential acceleration** (a_T). The only acceleration is a_c, the **centripetal acceleration**.



QUICK REVIEW. The **net force** on the bike will be the **difference** of the **downwards** and **upward forces** acting on the bike. The **upward force** is **F_N.** The **downwards force** is **F_w.**

$\sum F_{NET} = m \cdot a = m \cdot a_c = F_w - F_N$ (assuming down is +)

Since $a_c = \frac{v^2}{r}$ and $F_w = m \cdot g$, substituting:

$$m \cdot \frac{v^2}{r} = m \cdot g - F_N$$

What is the maximum velocity the bike could go so as to not lose contact with the hill? Assuming a circular hill, we make the contact force (F_N) between the hill and the bike minimum at this maximum speed. So

$$m \cdot \frac{v^2}{r} = m \cdot g - F_N$$

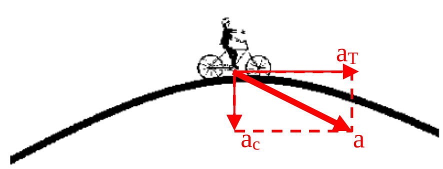
$$m \cdot \frac{v^2}{r} = m \cdot g - (0)$$

$$m \cdot \frac{v^2}{r} = m \cdot g$$

$$\frac{v^2}{r} = g$$

$$v = \sqrt{r \cdot g}$$

If the bike was *not* moving at **constant speed** around the circle, then the answer would have been C. since you would have not only the **centripetal acceleration**, a_c, but also the **tangential acceleration**, a_T.



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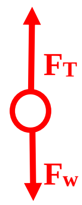
Base your answers to questions 4 and 5 on the following situation.

An object weighing 10 N swings at the end of a rope that is 0.72 m long as a simple pendulum. At the bottom of the swing, the tension in the string is 12 N.

4. ___**A.**___ What is the magnitude of the centripetal acceleration at the bottom of the swing)?

A) 2 m/s² D) 12 m/s²
B) 4 m/s² E) 22 m/s²
C) 10 m/s²



Ans. At the bottom of the swing the force diagram of the pendulum looks as shown in the Figure at right.

The **net force** on the pendulum bob will be the **difference** of the **downwards** and **upward forces** acting on the bob. The **upward force** is $\mathbf{F_T}$. The **downwards force** is $\mathbf{F_w}$.

$\sum \mathbf{F_{NET}} = \mathbf{m \cdot a} = \mathbf{m \cdot a_c} = \mathbf{F_w - F_T}$ (assuming down is +)

Solving for a_c and plugging in our given values:

$m \cdot a_c = F_w - F_T$

$a_c = \frac{F_w - F_T}{m}$

$a_c = \frac{10\ N - 12\ N}{1\ kg}$

$F_w = m \cdot g$
$10\ N = m \cdot (10\ m/s^2)$

$a_c = -2\frac{m}{s^2}$

The magnitude is just the absolute value of the answer, $|a_c|$ without the direction (–).

5. ___**A.**___ What is the speed of the object at the bottom of the swing?

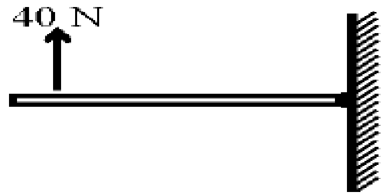
A) 0.6 m/s D) 2.4 m/s
B) 1.2 m/s E) 7.2 m/s
C) 2.0 m/s

Ans. The velocity can be obtained from the formula for the centripetal acceleration:

$a_c = \frac{v^2}{r} \rightarrow v = \sqrt{a_c \cdot r} = \sqrt{(2\frac{m}{s^2}) \cdot (0.72\ m)}$

$v = \sqrt{0.36} = 0.6$ m/s

Base your answers to questions **6** and **7** on the picture below, which represents a rigid uniform rod with a mass of 6 kg and a length of 1.0 m is pivoted on the right end. It is held in equilibrium by an upward force of 40 N.



6. ___**C.**___ How far from the left end of the rod should the force be placed to maintain equilibrium?

A) 10 cm D) 40 cm
B) 20 cm E) 50 cm
C) 25 cm

Ans. Since the rod is uniform, we can assume that its **center of mass** is at its geometric center.

Since the bar is 1.0 m long, the x_{cm} is at 0.5 m.

So we have a downward force of

$F\downarrow = m \cdot g = 6\ kg(10\ m/s^2) = 60\ N$ at 0.5 m away from the pivot point.

To balance this out we need

$\tau_c = \tau_{cc}$ (clockwise and counter-clockwise)

$r \times F = r \times F$

$r \times (40\ N) = (0.5\ m) \times (60\ N)$

$40\ r = 30$

$r = 0.75\ m$

From the *left* end, this is **0.25 m.**

7. ___**B.**___ What force is applied to the rod by the pivot?

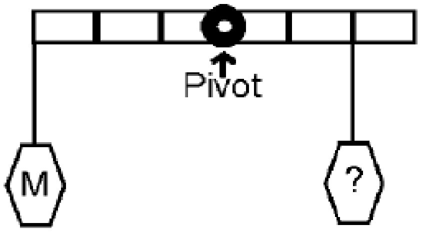
A) 10 N D) 60 N
B) 20 N E) 100 N
C) 40 N

Ans. We need to find the net force for equilibrium to exist (for the bar to be still) forgetting about torque for the moment.

$F_{NET} = F\downarrow - F\uparrow$

$F_{NET} = 60\ N - 40\ N =$ **20 N**

8. ___**D.**___ A uniform wooden board of mass 10 *M* is held up by a nail hammered into a wall. A block of mass *M* rests *L/2* away from the pivot. Another block of a certain mass is hung a distance *L/3*. The system is in static equilibrium.



What is the measure of the mass labeled "***?***" ?

A) $\frac{M}{2}$ D) **$\frac{3M}{2}$**
B) $\frac{M}{3}$ E) 2*M*
C) $\frac{M}{2}$

Ans. To balance this out we need

$\tau_c = \tau_{cc}$ (clockwise and counter-clockwise)

$r \times F = r \times F$

$(L/2) \times (M \cdot g) = (L/3) \times (M_2 \cdot g)$

$M \cdot g \cdot L/2 = M_2 \cdot g \cdot L/3$

$\frac{3MgL}{2gL} = M_2$

$\boldsymbol{\frac{3M}{2} = M_2}$

9. ___**B.**___ The angular velocity of a rotating disk with a radius of 2 m decreases from 6 rads per second to 3 rads per second in 2 seconds. What is the linear acceleration of a point on the edge of the disk during this time interval?

A) Zero D) 3/2 m/s^2
B) –3 m/s^2 E) 3 m/s^2
C) –3/2 m/s^2

Ans. We will use the two relationships:

$a_T = \alpha \cdot r$ and $\alpha = \frac{\omega - \omega_o}{t}$

$\alpha = \frac{\omega - \omega_o}{t} = \frac{3\frac{rad}{s} - 6\frac{rad}{s}}{2\,s} = \frac{-3}{2} = -1.5 \text{ rad/s}^2$

$a_T = \alpha \cdot r = (-1.5 \text{ rad/s}^2)(2 \text{ m}) = \mathbf{-3 \text{ m/s}^2}$

10. ___**E.**___ A solid sphere of radius 0.2 m and mass 2 kg is at rest at a height 7 m at the top of an inclined plane making an angle 60° with the horizontal. Assuming no slipping, what is the speed of the cylinder at the bottom of the incline?

A) Zero D) 6 m/s
B) 2 m/s **E) 10 m/s**
C) 4 m/s

Ans. $E_{in} = E_{out}$

$GPE_{in} = TKE_{out} + RKE_{out}$

$mgh = ½mv^2 + ½I\omega^2$

$mgh = ½mv^2 + ½\cdot(\frac{2}{5}\boldsymbol{m \cdot r^2})\left(\frac{v}{r}\right)^2$

$gh = ½v^2 + ½\cdot(\frac{2}{5}\cdot\boldsymbol{r^2})\frac{v^2}{r^2}$

$g \cdot h = ½v^2 + \frac{1}{5}v^2$

$g \cdot h = \frac{7}{10}v^2$

$\sqrt{\frac{10}{7}gh} = v$

$\sqrt{\frac{10}{7}(10\frac{m}{s^2})(7\text{ m})} = v$

$\sqrt{100\,m} = v$

10 m/s = v

11. ___**B.**___ A spinning object with moment of inertia *I* increases in angular speed from $\omega = 0$ to ω_a in *t* seconds. What is the average power delivered to the object during this interval *t*?

A) $I\omega_a/2t^2$
B) $I\omega_a^2/t$
C) $I\omega_a^2/2t$
D) $I\omega_a^2/t^2$
E) $I\omega_a^2/2t^2$

Ans. We will have to use the formula for power in rotational motion $P_{rot} = \tau \bullet \omega$ and the formula for torque $\Sigma\tau = I \cdot \alpha$

$P_{rot} = \tau \bullet$

$\omega = I \cdot \alpha \cdot \omega = I\left(\frac{\omega - \omega_o}{t}\right) \cdot \omega = I\left(\frac{\omega_a - 0}{t}\right) \cdot \omega_a = I \cdot \frac{\omega_a^2}{t}$

12. __**?**____ What is the moment of inertia of a spinning object of radius 0.5 m and mass 6 kg moving at 5 m/s, if it has a kinetic energy of 100 J?

A) 1 kg·m^2 D) 8 k·gm^2
B) 2 kg·m^2 E) 20 kg·m^2
C) 4 kg·m^2

Ans. $TKE_{combined} = TKE + RKE$

$TKE_{combined} = ½mv^2 + ½I\omega^2$

$TKE_{combined} = ½mv^2 + ½I\left(\frac{v}{r}\right)^2$

$100 \text{ J} = ½(6 \text{ kg})(5 \text{ m/s})^2 + ½I\left(\frac{5\frac{m}{s}}{0.5\,m}\right)^2$

$100 = 75 + 50I$

$25 = 50I$

½ = I

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13. ___**B.**___ Which of the following objects has the least kinetic energy at the bottom of the incline if they all have the same mass and radius?
A) cylinder
B) sphere
C) hoop
D) all have the same
E) not enough information

Ans. the sphere has the most mass concentrated the closest to its **axis of rotation** ($I = 2/5mr^2$). This means it has the greatest **inertia** and thus the greatest **rotational kinetic energy** ($RKE = ½I\omega^2$) all other things being equal.

14. ___**C.**___ Which of the following objects has the greatest rotational kinetic energy at the bottom of the incline if they all have the same mass & radius?
A) cylinder
B) sphere
C) hoop
D) all have the same
E) not enough information

Ans. the hoop has the most mass concentrated the furthest away from the **axis of rotation** ($I = mr^2$). This means it has the greatest **inertia** and thus the greatest **rotational kinetic energy** ($RKE = ½I\omega^2$) all other things being equal.

15. ___**C.**___ A solid cylinder of radius .2 m and mass 2 kg is at rest at a height 1.2 m at the top of an inclined plane making an angle 60° with the horizontal. Assuming no slipping, what is the speed of the cylinder at the bottom of the incline?
A) Zero
B) 2 m/s
C) 4 m/s
D) 6 m/s
E) 10 m/s

Ans.
$$E_{in} = E_{out}$$
$$GPE_{in} = TKE_{out} + RKE_{out}$$
$$mgh = ½mv^2 + ½I\omega^2$$
$$mgh = ½mv^2 + ½\cdot\left(\frac{1}{2}\boldsymbol{m}\cdot\boldsymbol{r}^2\right)\left(\frac{v}{r}\right)^2$$
$$gh = ½v^2 + ½\cdot\left(\frac{1}{2}\cdot\boldsymbol{r}^2\right)\frac{\boldsymbol{v}^2}{\boldsymbol{r}^2}$$
$$g\cdot h = ½v^2 + \frac{1}{4}v^2$$
$$g\cdot h = \frac{3}{4}v^2$$
$$\sqrt{\frac{4}{3}gh} = v$$
$$\sqrt{\frac{4}{3}\left(10\,\frac{\text{m}}{\text{s}^2}\right)(1.2\text{ m})} = v$$
$$\sqrt{\frac{4}{3}(10)\left(\frac{6}{5}\right)} = v$$
$$\sqrt{16} = v$$
$$\mathbf{4\ m/s = v}$$

16. ___**B.**___ What is the ratio of the moment of inertia of a cylinder of mass *m* and radius *r* to the moment of inertia of a hoop of the same mass and same radius?
A) 1:1
B) 1:2
C) 2:1
D) 1:4
E) 4:1

Ans. $\frac{I_{cylinder}}{I_{hoop}}$
$$\frac{\frac{1}{2}\boldsymbol{m}\cdot\boldsymbol{r}^2}{\boldsymbol{m}\cdot\boldsymbol{r}^2}$$
$$\frac{\frac{1}{2}}{1} = \frac{\mathbf{1}}{\mathbf{2}}$$

17. ___**E.**___ A 4 kg object moves in a circle of radius 8 m at a constant speed of 2 m/s. What is the angular momentum of the object with respect to an axis perpendicular to the circle and through its center?
A) 2 N•s
B) 6 N•m/kg
C) 12 kg•m/s
D) 24 m^2/s
E) 64 kg•m^2/s

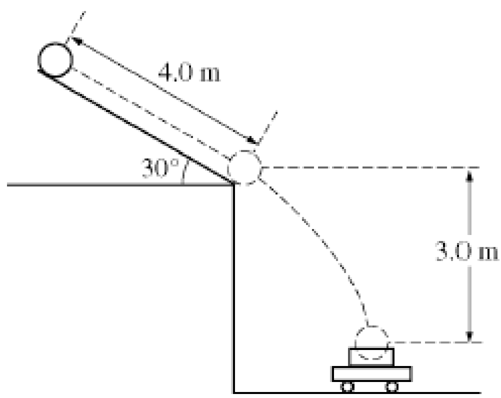
Ans. $\mathbf{L = r \times p = r \times m\cdot v = 8\ m \cdot 4\ kg \cdot 2\ m/s}$
$= \mathbf{64}$ kg•m^2/s

18. ___**E.**___ A solid cylinder with diameter 20 cm has an angular velocity of 10 rad/s and angular momentum of 2 kg·m^2/s. What is its mass?
A) 0.1 kg
B) 1 kg
C) 2 kg
D) 5 kg
E) 10 kg

Ans. $\mathbf{L = I\,\omega \rightarrow L = \frac{1}{2}\boldsymbol{m}\cdot\boldsymbol{r}^2\cdot\omega \rightarrow}$

$$\mathbf{2\ kg\cdot m^2/s = \frac{1}{2}m\cdot(0.20\ m)^2\cdot(10\ rad/s)}$$
$$2 = 0.2m$$
$$\mathbf{10\ kg = m}$$

FREE RESPONSE 1

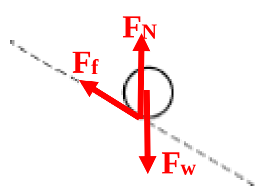


Note: Figure not drawn to scale.

Mech. 2.

A bowling ball of mass 6.0 kg is released from rest from the top of a slanted roof that is 4.0 m long and angled at 30°, as shown above. The ball rolls along the roof without slipping. The rotational inertia of a sphere of mass M and radius R about its center of mass is $\frac{2}{5}MR^2$.

(a) On the figure below, draw and label the forces (not components) acting on the ball at their points of application as it rolls along the roof.



(b) Calculate the force due to friction acting on the ball as it rolls along the roof. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

(c) Calculate the linear speed of the center of mass of the ball when it reaches the bottom edge of the roof.

(d) A wagon containing a box is at rest on the ground below the roof so that the ball falls a vertical distance of 3.0 m and lands and sticks in the center of the box. The total mass of the wagon and the box is 12 kg. Calculate the horizontal speed of the wagon immediately after the ball lands in it.

(b) Ans.

The forces along the plane are the **component of the weight along the plane** F_x and the **force of friction**, F_f.

$F_{net} = ma = F_x - F_f = mg \sin\theta - F_f$

Before rotational motion, we could define the force of friction F_f strictly in terms of linear variables. But it is the friction that is making the sphere roll. Let's take a look at friction F_f and its role in making the ball roll down the incline.

torque is given by $\tau = I \cdot \alpha$

Torque is also $F \times r$ where the force F that is making the ball spin as it is in contact with the incline is the frictional force, F_f.

Substituting: $F_f \times r = I \cdot \alpha$

Since the ball is spherical, we can replace I with $\frac{2}{5}mr^2$, the formula for ***the moment of inertia.***

We can also replace α since $a = \alpha r$, where a is the tangential acceleration of the ball and r is the radius of the sphere.

$$F_f \times r = \left(\frac{2}{5}mr^2\right)\cdot\frac{a}{r}$$

Solving for F_f: $F_f = \left(\frac{2}{5}mr^2\right)\cdot\frac{a}{r^2} = \frac{2}{5}ma \rightarrow F_f = \frac{2}{5}m\cdot a$

If we solve for $m\cdot a$, we get $\frac{5}{2}F_f = m\cdot a$

We now have two expressions for m·a: $\frac{5}{2}F_f = m\cdot a$ and $m\cdot a = m\cdot g\sin\theta - F_f$

Setting them equal to each other: $\frac{5}{2}F_f = m\cdot g\sin\theta - F_f$

Solving for F_f: $\frac{7}{2}F_f = m\cdot g\sin\theta \rightarrow F_f = \frac{2}{7}m\cdot g\sin\theta \rightarrow F_f = \frac{2}{7}(6\text{ kg})\cdot(10\text{ m/s}^2)\sin(30°) =$ ***8.6 N***

(c) Ans. To find the **speed** of the sphere we have to use our **conservation of energy** relationship.

$$E_{in} = E_{out}$$

$$GPE_{in} = TKE_{out} + RKE_{out}$$

$$mgh = ½mv^2 + ½I\omega^2$$

$$m\cdot g\cdot d\cdot\sin\theta = ½mv^2 + ½\cdot\left(\frac{2}{5}\boldsymbol{m\cdot r^2}\right)\left(\frac{v}{r}\right)^2$$

$$g\cdot d\cdot\sin\theta = ½v^2 + ½\cdot\left(\frac{2}{5}\cdot\boldsymbol{r^2}\right)\frac{v^2}{r^2}$$

$$g\cdot d\cdot\sin\theta = ½v^2 + \frac{1}{5}v^2$$

$$g\cdot d\cdot\sin\theta = \frac{7}{10}v^2$$

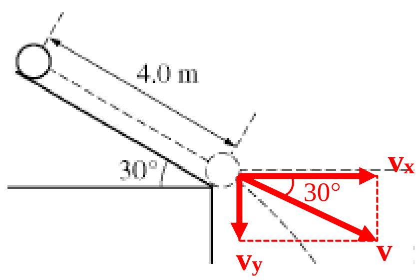
$$\sqrt{\frac{10}{7}g\cdot d\cdot\sin\theta} = v$$

$$\sqrt{\frac{10}{7}\left(10\frac{m}{s^2}\right)(4\text{ m})\cdot\sin(30°)} = v$$

5.34 m/s = v

(d) Ans. The **horizontal velocity** right as the ball comes rolling off the roof would be $\mathbf{v_x = v\cos 30°}$ (see the Figure at right)

The ball will maintain this horizontal velocity in the absence of a horizontal acceleration. This is the velocity that will make the wagon move forward.



The momentum before and after the collision must be conserved.

$$p_{in} = p_{fi}$$

$$m_1\cdot v_{in} = m_2\cdot v_{fi}$$

$$m_1\cdot v_x = m_2\cdot v_{fi}$$

$$m_1\cdot v\cos 30° = m_2\cdot v_{fi}$$

$$\frac{m_1}{m_2}\cdot v\cos 30° = v_{fi}$$

$$\frac{6\,kg}{18\,kg}\cdot\left(5.34\frac{m}{s}\cos 30°\right) = v_{fi}$$

1.54 m/s = v_{fi}

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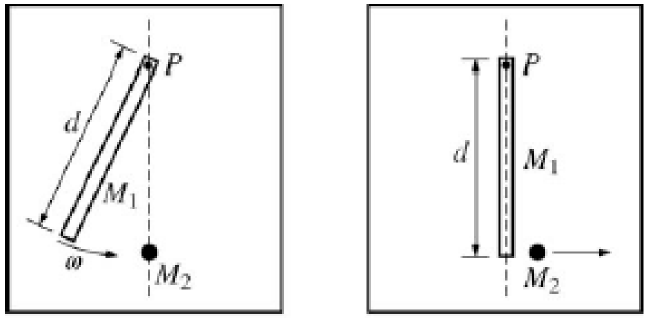
FREE RESPONSE 2



Before Collision   After Collision

TOP VIEWS

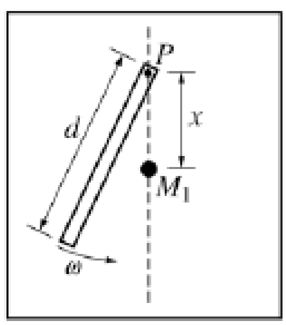
Mech. 3.

A system consists of a ball of mass M_2 and a uniform rod of mass M_1 and length d. The rod is attached to a horizontal frictionless table by a pivot at point P and initially rotates at an angular speed ω, as shown above left. The rotational inertia of the rod about point P is $\frac{1}{3}M_1d^2$. The rod strikes the ball, which is initially at rest. As a result of this collision, the rod is stopped and the ball moves in the direction shown above right. Express all answers in terms of M_1, M_2, ω, d, and fundamental constants.

(a) Derive an expression for the angular momentum of the rod about point P before the collision.

(b) Derive an expression for the speed v of the ball after the collision.

(c) Assuming that this collision is elastic, calculate the numerical value of the ratio M_1/M_2.



Before Collision

(d) A new ball with the same mass M_1 as the rod is now placed a distance x from the pivot, as shown above. Again assuming the collision is elastic, for what value of x will the rod stop moving after hitting the ball?

(a) **Ans.** The **angular momentum** of this rigid uniform body is given by **L** = I·ω = $(\frac{1}{3}M_1d^2)\cdot\omega$ **since the** given **rotational inertia** is **I** = $\frac{1}{3}M_1d^2$.

(b) The **momentum** will be conserved **after** the impact.

$$L_{in} = L_{fi}$$

$$\left(\frac{1}{3}M_1d^2\right)\cdot\omega = M_2\cdot v\cdot d$$

$$\frac{\frac{1}{3}M_1 d^2\omega}{M_2 d} = v$$

(c) If the collision is elastic then the kinetic energy is also conserved. This means we have a second relationship between M_1 and M_2: two equations and two unknowns.

$$E_{in} = E_{out}$$
$$RKE_{in} = TKE_{out}$$
$$\tfrac{1}{2}I\cdot\omega^2 = \tfrac{1}{2}mv^2$$
$$\left(\frac{1}{3}M_1 d^2\right)\cdot\omega^2 = M_2\left(\frac{\frac{1}{3}M_1 d^2\omega}{M_2 d}\right)^2$$
$$\frac{1}{3}M_1 d^2\cdot\omega^2 = M_2\cdot\frac{\frac{1}{9}M_1^2 d^4\omega^2}{M_2^2 d^2}$$
$$\frac{1}{3}M_1 d^2\cdot\omega^2 = \frac{M_1^2 d^2\omega^2}{9M_2}$$
$$M_1 = \frac{M_1^2 d^2\omega^2}{3M_2 d^2\cdot\omega^2}$$
$$M_1 = \frac{M_1^2}{3M_2}$$
$$\frac{M_1\cdot M_2}{M_1^2} = \frac{1}{3}$$
$$\frac{M_2}{M_1} = \frac{1}{3} \quad \text{so} \quad \frac{M_1}{M_2} = \frac{3}{1}$$

(d) The **momentum** will be conserved **after** the impact.

$$L_{in} = L_{fi}$$
$$I\cdot\omega = p\cdot r$$
$$\left(\frac{1}{3}M_1 d^2\right)\cdot\omega = M_1\cdot v\cdot x$$

Solving for **v as** we did in **Probl. (a):**

$$\frac{\frac{1}{3}M_1 d^2\omega}{M_1 x} = v$$

The masses cancel out.

$$\frac{\frac{1}{3}d^2\omega}{x} = v$$

Since the collision is **elastic,** we can call upon the **kinetic energy** relationship as well as in Probl. (c):

$$E_{in} = E_{out}$$
$$RKE_{in} = TKE_{out}$$
$$\tfrac{1}{2}I\cdot\omega^2 = \tfrac{1}{2}mv^2$$
$$\left(\frac{1}{3}M_1 d^2\right)\cdot\omega^2 = M_1\left(\frac{\frac{1}{3}d^2\omega}{x}\right)^2$$

Simplifying and solving for ***x*:**

$$\frac{1}{3}M_1 d^2\cdot\omega^2 = M_1\cdot\frac{\frac{1}{9}d^4\omega^2}{x^2}$$
$$\frac{1}{3}d^2\cdot\omega^2 = \frac{\frac{1}{9}d^4\omega^2}{x^2}$$
$$\frac{1}{3}x^2 = \frac{\frac{1}{9}d^4\omega^2}{\omega^2 d^2}$$
$$\frac{1}{3}x^2 = \frac{1}{9}d^2$$
$$x^2 = \frac{1}{3}d^2$$
$$x = \frac{\sqrt{3}}{3}d$$

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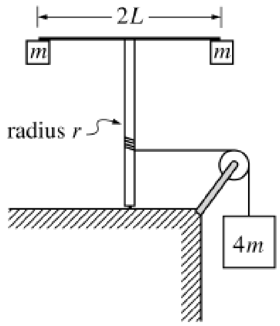
FREE RESPONSE 3



Experiment A

Mech 3.

A light string that is attached to a large block of mass $4m$ passes over a pulley with negligible rotational inertia and is wrapped around a vertical pole of radius r, as shown in Experiment A above. The system is released from rest, and as the block descends the string unwinds and the vertical pole with its attached apparatus rotates. The apparatus consists of a horizontal rod of length $2L$, with a small block of mass m attached at each end. The rotational inertia of the pole and the rod are negligible.

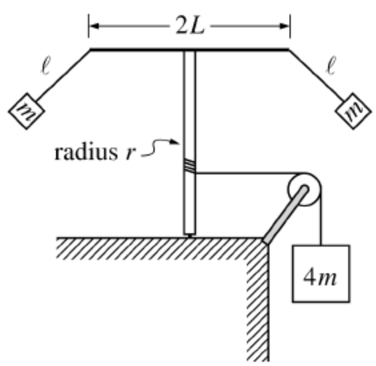
(a) Determine the rotational inertia of the rod-and-block apparatus attached to the top of the pole.

(b) Determine the downward acceleration of the large block.

(c) When the large block has descended a distance D, how does the instantaneous total kinetic energy of the three blocks compare with the value $4mgD$? Check the appropriate space below.

____ Greater than $4mgD$ ____ Equal to $4mgD$ ____ Less than $4mgD$

Justify your answer.



Experiment B

The system is now reset. The string is rewound around the pole to bring the large block back to its original location. The small blocks are detached from the rod and then suspended from each end of the rod, using strings of length ℓ. The system is again released from rest so that as the large block descends and the apparatus rotates, the small blocks swing outward, as shown in Experiment B above. This time the downward acceleration of the block decreases with time after the system is released.

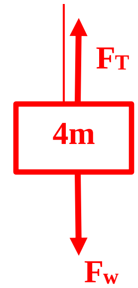
(d) When the large block has descended a distance D, how does the instantaneous total kinetic energy of the three blocks compare to that in part (c) ? Check the appropriate space below.

____ Greater ____ Equal ____ Less

Justify your answer.

(a) Ans. The **rotational inertia** is given by $I = \Sigma mr^2$.

$I = m \cdot L^2 + m \cdot L^2 = \mathbf{2m \cdot L^2}$



(b) The acceleration of the downward block can be obtained from $F_{NET} = m \cdot a = 4m \cdot a = F_w - F_T$

$4m \cdot a = 4m \cdot g - F_T$

Solving for F_T: $F_T = 4m \cdot g - 4m \cdot a$

The **tension** is related to the **torque** that the rod experiences.

torque is given by $\tau = I \cdot \alpha$

Torque is also $F \times r$ where the force F that is making the rod spin is equal to F_T, the tension in the rope.

Substituting: $F_T \times r = I \cdot \alpha$

In Probl. (a), we found that ***the moment of inertia, I = 2m·L²*** for the rod-and-block apparatus.

We can also replace α since $a = \alpha r$, where a is the tangential acceleration of the rod-and-block apparatus and r is equal to L.

$$F_T \times r = (2mL^2) \cdot \frac{a}{r}$$

Solving for F_T: $F_T \times r = (2mL^2) \cdot \frac{a}{r}$

$$F_T = \frac{2mL^2 a}{r^2}$$

Setting the tensions equal to each other and solving for a:

$$\frac{2mL^2 a}{r^2} = 4m \cdot g - 4m \cdot a$$

$$\frac{2mL^2 a}{r^2} + 4m \cdot a = 4m \cdot g$$

Canceling out the mass ***m*:**

$$\frac{2L^2 a}{r^2} + 4a = 4g$$

$$\mathbf{a}\left(\frac{2L^2}{r^2} + 4\right) = 4g$$

$$\mathbf{a}\left(\frac{2L^2 + 4r^2}{r^2}\right) = 4g$$

$$\mathbf{a} = \frac{4gr^2}{2L^2 + 4r^2}$$

$$\mathbf{a} = \frac{2gr^2}{L^2 + 2r^2}$$

(c) The **total kinetic energy,** by **conservation of energy,** must be **equal to** the **loss of potential energy of the falling block** of mass **4m:**

$\mathbf{GPE_{in} = 4mgD = TKE_{fi}}$

(d) The **total kinetic energy,** by **conservation of energy,** must be **less than** the **loss of potential energy of the falling block** of mass **4m** because some of the energy has gone into **raising** the **two spinning masses** on the rod-and-block apparatus. They now have some **GPE.**

$GPE_{in} = 4mgD = TKE_{fi} + \mathbf{GPE_{fi}}$